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Analysis of Liquid and Structural Transients in Piping by the Method of Characteristics

Since liquid-filled piping systems are composed of slender elements, their transient behavior can be described as one-dimensional wave phenomena. Seven wave components are identified: coupled axial compression of liquid and pipe material; coupled transverse shear and bending of the pipe elements in two principal directions; and torsion of the pipe wall. Utilizing the method of characteristics, the combined system of difference equations for pipe elements and the pipe junction boundary conditions comprise a general mathematical tool for predicting the liquid pressure and pipe stress responses to transient excitation of either liquid or piping. The complexity of fluid-structure interaction that can take place is demonstrated.

Introduction

The analytical procedures usually employed for design of piping systems are approximate, in part since they neglect the effects of structural mass, stiffness and damping on transient pressures in the contained liquid. Although a number of studies addressing this issue have appeared in the recent literature, and although significant gains have been realized, there remains a need to rigorously investigate all of the mechanisms by which properties of the pipe structure influence liquid transients and vice versa.

In their attempts to more accurately couple fluid-structure interaction in piping systems, investigators have utilized several different methodologies. The conventional and most common means to represent the structural motion of the piping has been to employ finite element techniques, and the coupled liquid motion has been modeled either by finite elements or by the method of characteristics. Complete modeling has not yet been achieved from the viewpoint of correctly coupling the significant motions in both the piping and the liquid. This paper presents wave equations which account for the coupled motion of piping and contained liquid. A technique based on acoustic wave analysis allows explicit solution of the system of equations.

Wave Equations for Fluid-Structure Interaction

The primary coupling mechanisms for fluid-structure interaction in liquid-filled piping are: Poisson coupling, by which dynamic pressure and the resulting circumferential strain induce axial strain in the pipe wall; and junction coupling, by which dynamic pressure exerts an axial resultant on piping and, conversely, pipe motion generates changes in dynamic pressure. The former mechanism occurs throughout the length of a pipe and the latter is associated with junctions where flow area or direction changes.

A number of investigators [1-5] have identified axial stress waves in the pipe wall which are generated by the Poisson effect. In addition, a few others [6-8] have introduced shear, bending and torsion waves in the structural elements. Most notably, Wilkinson [6, 7] clearly shows the interaction between liquid and structure at pipe fittings and the manner in which the various waves are generated. His analytical techniques were based on a frequency-domain formulation [6] or on a simplified wave approach [7], which impose limitations on the excitation function and complexity of the piping system.

Fundamental Equations. Since liquid-filled piping systems are composed of slender components, their transient behavior can be described as one-dimensional wave phenomena. Consider the prismatic liquid-filled pipe element shown in Fig. 1. It can transmit torsional waves in the pipe wall, transverse shear and bending waves in the pipe wall, and axial compression waves in both the pipe wall and the liquid. The equations to describe such motion are given below.

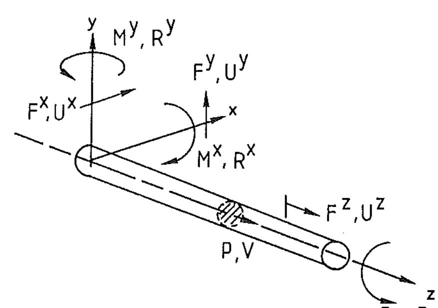

Fig. 1 Pipe element

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1) Torsion in the pipe wall:

$$M_z^z - \rho_p J R_t^z = 0 \quad (1)$$

$$M_t^z - GJR_z^z = 0 \quad (2)$$

2) Shear and bending in the *y-z* plane [9]:

$$M_z^x - (\rho_p I_p + \rho_f I_f) R_t^x - F^y = 0 \quad (3)$$

$$M_t^x - EI_p R_z^x = 0 \quad (4)$$

$$F_z^y - (\rho_p A_p + \rho_f A_f) U_t^y = 0 \quad (5)$$

$$F_t^y - GA_p (U_z^y + R^x) = 0 \quad (6)$$

3) Shear and bending in the *x-z* plane [9]:

$$M_z^y - (\rho_p I_p + \rho_f I_f) R_t^y + F^x = 0 \quad (7)$$

$$M_t^y - EI_p R_z^y = 0 \quad (8)$$

$$F_z^x - (\rho_p A_p + \rho_f A_f) U_t^x = 0 \quad (9)$$

$$F_t^x - GA_p (U_z^x - R^y) = 0 \quad (10)$$

4) Axial stress, pressure and velocities [1]:

$$P_z + \rho_f V_t = 0 \quad (11)$$

$$P_t + K^* V_z - 2\nu K^* U_z^z = 0; \quad K^* = \frac{K}{1 + \frac{2rK}{eE}(1 - \nu^2)} \quad (12)$$

$$F_z^z - A_p \rho_p U_t^z = 0 \quad (13)$$

$$F_t^z - A_p E U_z^z - \frac{r\nu A_p}{e} P_t = 0 \quad (14)$$

The formulations are based on assumptions of linear elasticity, no buckling, cylindrical pipes, negligible radial inertia, and low Mach number, i.e., small liquid velocity relative to acoustic velocity. Furthermore, it is understood that instantaneous pressures will remain above vapor pressure, so that no cavitation will occur, and that high-frequency lobar modes of the pipe are not excited by the dynamic interaction of the fluid and structure.

Numerical Model

Characteristic Equations. Equations (1)–(14) are hyperbolic partial differential equations and can be converted to ordinary differential equations by the method of characteristics transformation [10]. The method has been applied to Timoshenko beam theory by Leonard and Budiansky [11], and to axial coupled behavior for liquid-filled pipe by Otwell, Wiggert and Hatfield [1]. The complete set of characteristic and compatibility equations corresponding to equations (1)–(14) is given below. Equations 20 and 21 have been simplified by assuming that

$$\nu^2 << \frac{e}{2r}\left(\frac{E}{K^*} + \frac{\rho_p}{\rho_f}\right).$$

$$\frac{dM^z}{dt} - a\rho_p J \frac{dR^z}{dt} = 0 \quad (15a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{G/\rho_p} \quad (15b)$$

$$\frac{dM^x}{dt} - a(\rho_p I_p + \rho_f I_f)\frac{dR^x}{dt} - aF^y = 0 \quad (16a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{\frac{EI_p}{\rho_p I_p + \rho_f I_f}} \quad (16b)$$

$$\frac{dF^y}{dt} - a(\rho_p A_p + \rho_f A_f)\frac{dU^y}{dt} - GA_p R^x = 0 \quad (17a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{\frac{GA_p}{\rho_p A_p + \rho_f A_f}} \quad (17b)$$

$$\frac{dM^y}{dt} - a(\rho_p I_p + \rho_f I_f)\frac{dR^y}{dt} + aF^x = 0 \quad (18a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{\frac{EI_p}{\rho_p I_p + \rho_f I_f}} \quad (18b)$$

$$\frac{dF^x}{dt} - a(\rho_p A_p + \rho_f A_f)\frac{dU^x}{dt} + GA_p R^y = 0 \quad (19a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{\frac{GA_p}{\rho_p A_p + \rho_f A_f}} \quad (19b)$$

$$\frac{dF^z}{dt} - a\rho_p A_p \frac{dU^z}{dt} - 2\nu A_f \frac{dP}{dt} = 0 \quad (20a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{E/\rho_p} \quad (20b)$$

$$\frac{dV}{dt} + \frac{a}{K^*}\frac{dP}{dt} + \frac{2\rho_p \nu}{\rho_f\left(\frac{E}{K^*} - \frac{\rho_p}{\rho_f}\right)}\frac{dU^z}{dt} - \frac{2a\nu}{K^* A_p\left(\frac{E}{K^*} - \frac{\rho_p}{\rho_f}\right)}\frac{dF^z}{dt} = 0 \quad (21a)$$

$$\frac{dz}{dt} = a = \pm\sqrt{K^*/\rho_f} \quad (21b)$$

Nomenclature

A = cross-sectional area
a = wave speed
B = coefficients for equation (22)
E = modulus of elasticity
e = thickness of pipe wall
F = force
G = shear modulus of rigidity
H = constant terms for equation (22)
I = moment of inertia
J = polar moment of inertia
K = bulk modulus of elasticity
M = moment
n = number of pipes meeting at junction
P = pressure
R = rotational velocity
r = radius of pipe cross-section
T = direction cosines
U = velocity of pipe
V = velocity of liquid
ρ = mass density
ν = Poisson's ratio

Superscripts

x,y,z = principal directions
T = matrix transposition

Subscripts

f = fluid
j = junction quantities in global coordinates
p = pipe
t = partial derivative with respect to time
z = partial derivative with respect to axial direction.

Note that distributed coupling takes place between shear and bending parameters in the compatibility relations equations (16*a*) and (17*a*), and equations (18*a*) and (19*a*). An additional coupling occurs between the axial pipe stress and velocity and the liquid pressure in Equations (20*a*) and (21*a*).

Boundary Conditions and Solution of System Equations. At the end of a given pipe element only one compatibility equation for a companion pair of variables is available for each adjoining pipe element. An additional equation in the form of a boundary condition is required. For example, in Fig. 2, equation (15) is available along the positive characteristic for computation of the torque M^z and the rotational velocity R^z. If the end of the element is fixed against twist, the boundary condition will be $R^z = 0$.

A junction is a point where several pipe ends are connected so that flow, forces and moments can be transmitted from pipe to pipe; the junction has no length or mass. At the junction of n prismatic pipes, $7(2n+1)$ equations can be formed which relate the continuity and equilibrium of the various displacements, pressures, forces and moments in the connecting pipes at the junction. These relations are combined with the finite difference approximations derived from equations (15)-(21), and by successive substitution a matrix equation with seven unknowns is developed:

$$\begin{Bmatrix} V_j \\ \{F_j\} \\ \{M_j\} \end{Bmatrix} = \sum_n \pm [T]^T[B][T] \begin{Bmatrix} P_j \\ \{U_j\} \\ \{R_j\} \end{Bmatrix} + \sum_n \pm [T]^T\{H\} \tag{22}$$

The vector $\{H\}$ contains known quantities, including forces, moments, pressures and velocities for previous times. The contents of $[B]$ are material properties and geometrical information that are time invariant. At a junction, either velocity V_j or pressure P_j will be known and the other quantity unknown. Similarly, unknown external forces F_j and moments M_j, that is reactions, will correspond to known translation and rotational velocities. Equation (22) may be partitioned and solved by inverting a portion of $\Sigma[T]^T[B][T]$. Inversion is performed only once for each junction, and is not required at each time step.

The method of characteristics formulation provides three useful features: 1) the Poisson coupling (of equations (20) and (21)) is represented; 2) nonlinearities such as fluid friction and material damping can be represented by additional terms in the compatibility equations; and 3) nonlinear boundary conditions such as pump trip, column separation, and external viscous or viscoelastic damping of pipe supports can be incorporated.

In the developed numerical scheme, pipe segments may be subdivided into smaller reaches if necessary. Equation (22) is applied at every junction and boundary location. The five characteristic lines identified with each pipe reach are projected back in time to an opposite junction, and time-line interpolations are utilized to obtain numerical values of the required parameters. With this scheme, two sources of error have been identified: numerical attenuation and phase changes due to the time-line interpolations [12], and inexact integration of the nondifferential shear and bending terms in the compatibility relations, Equations (16)-(19).

Application

The formulation is demonstrated for a system of three pipes directed orthogonally and connected in series as shown in Fig. 3. The piping is made of copper with mitered bends and an inside diameter of 25 mm; each reach is 2 m long. The conveyed liquid is water, and damping is neglected in the fluid as well as in the pipe structure. The upstream end of the piping (location A) is restrained from motion and connects to a reservoir with static pressure. The piping is constrained at the downstream end (location D) by connection to a valve. The system is excited by closure of the valve; initially the velocity is 1.0 m/s and subsequently the velocity decreases linearly to zero in approximately 2.2 ms. It is assumed that the static pressure is of sufficient magnitude that dynamic pressures will not reach vapor pressure.

Figures 4 through 6 are calculated responses to the valve closure excitation. In the figures, only the dynamic components are shown; total pressures, forces and moments may be computed as the sum of steady-state – that is, static – and dynamic components. The maximum permissible time incre-

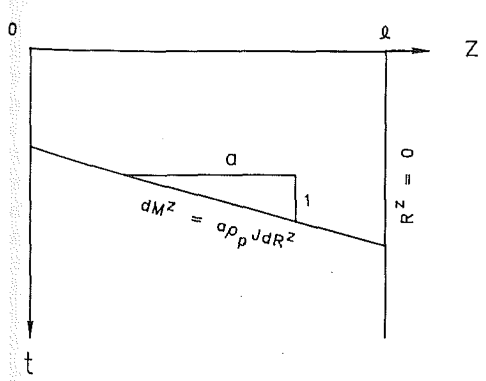

Fig. 2 Boundary condition with characteristics grid

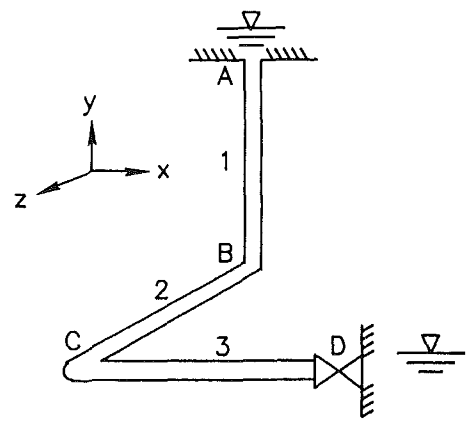

Fig. 3 Example piping

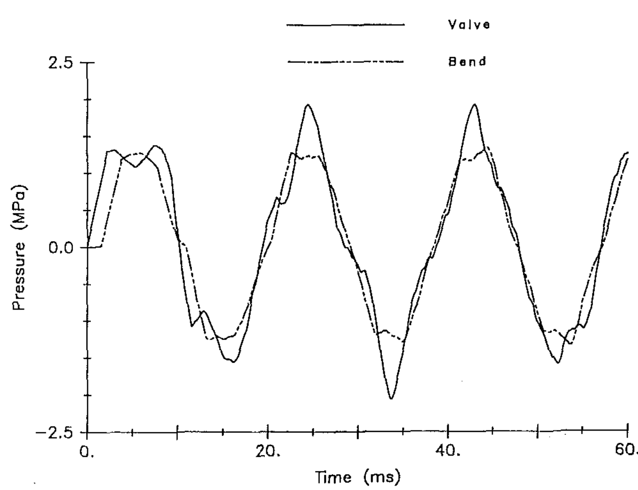

Fig. 4 Fluid pressures at bend (location C) and valve (location D)

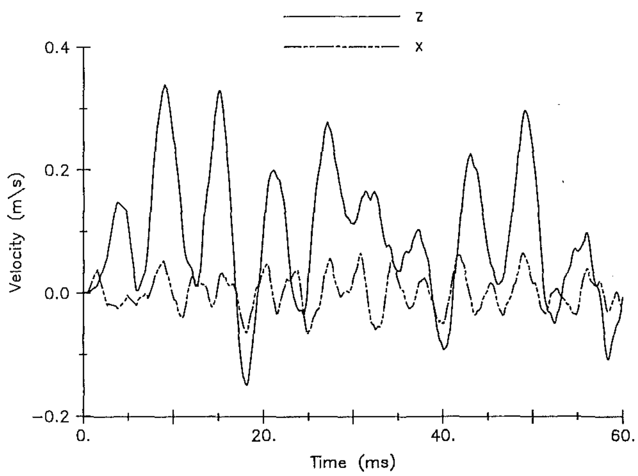

Fig. 5 Pipe velocities in *x* and *z* directions at bend (location C)

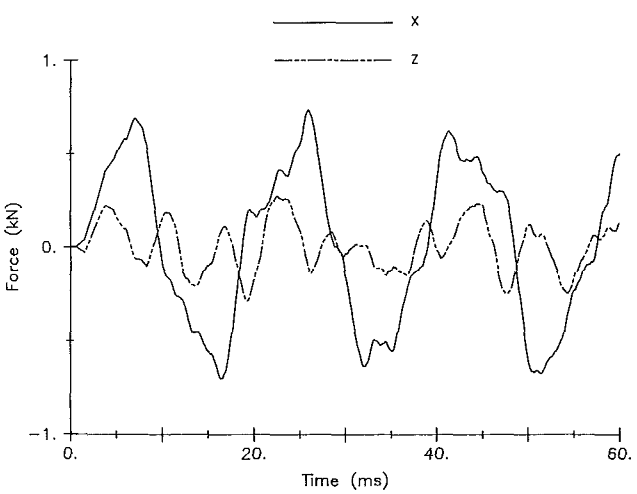

Fig. 6 Forces in *x* and *z* directions in pipe 3 at bend (location C)

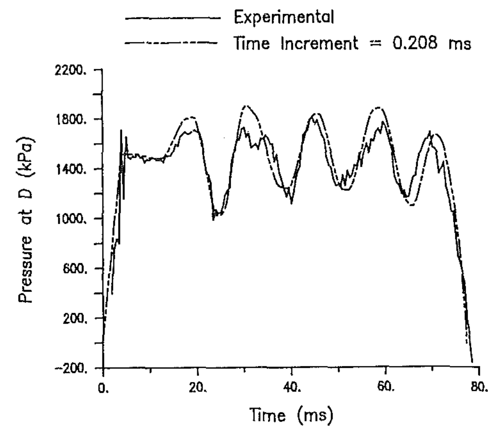

Fig. 7 Predicted and experimental fluid pressure at valve (location D)

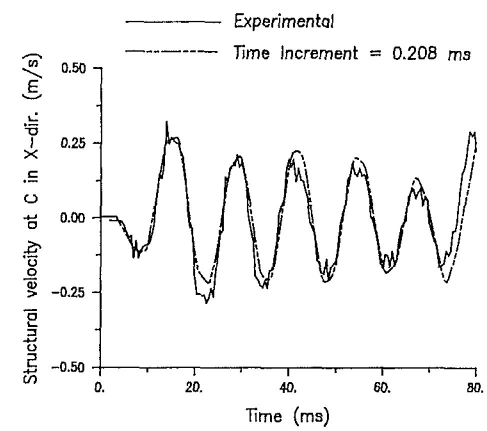

Fig. 8 Predicted and experimental structural velocity at bend (location C)

ment is computed as the pipe element length divided by the largest wave speed, i.e., 0.55 ms.

The effects of Poisson and junction coupling are apparent in Figs. 4 and 5. When the valve begins to close, the pressure at *D* increases causing the pipe to dilate. Because of the Poisson effect, the dilation generates an axial force. This force propagates through the pipe wall and reaches the bend at location *C* at the end of the first time interval, causing the bend to accelerate in the positive *x*-direction. Because of junction coupling, this motion causes as pressure rise at *C*, as well as motion in the positive *z*-direction. Pressure waves travel 2 m in three time intervals. In response to the primary pressure pulse, the motion of *C* in the *x*-direction reverses at the end of the third time interval. At the end of the fourth interval, the secondary pressure induced by the motion of bend *C* reaches location *D*, causing the pressure there to continue to rise after the valve is completely closed. Figure 6 illustrates the axial force and transverse shear force in the *z*-direction for the end of pipe 3 at junction *C*.

A second configuration of the piping shown in Fig. 3 is used to simulate the laboratory experiment described in Wiggert et al. [1]. Elbow *B* is constrained from motion and elbow *C* is free to move in the *x-z* plane. Pipe segments 1, 2 and 3 are respectively 28.0 m, 7.6 m, and 12.3 m in length. Significant structural motion is to be expected in segments 2 and 3. Predicted pressure at location D and predicted structural velocity in the *x*-direction at location C are presented along with experimental data in Figs. 7 and 8, respectively. For the computation, the piping was divided into seven links: five for segment 1, one each for segments 2 and 3. A time increment of 0.2081 ms was employed.

The numerical model can be seen to predict the motion and pressure in an acceptable fashion. However, no significant shear and bending moments were developed in the experiment, so that the numerical model is not severely taxed with regard to accurately predicting those parameters.

Summary and Conclusions

Because piping systems are composed primarily of slender components, transient responses may be represented by one-dimensional wave phenomena. Seven wave components must be considered: pressure in the liquid; and in the pipe material, axial compression, torsional shear, transverse shear in two orthogonal directions, and bending about the two transverse axes. For linearly elastic pipe material, torsion and transverse shear are not coupled, nor are bending and axial compression. The phenomenon of axial compression of liquid and pipe material consists of two distinct but coupled waves. Timoshenko beam theory [9] describes the interaction of transverse shear and bending of the pipe elements. The various independent wave mechanisms are coupled at pipe junctions.

Propagation of the waves along a straight pipe may be described by finite difference equations derived from the method of characteristics. The combined system of difference equations for pipe elements and the junction boundary conditions comprise a general mathematical tool for predicting the liquid pressure and pipe stress responses to transient excitation of either liquid or piping. Two possible sources of error in the numerical solution scheme have been identified; additional development is necessary before the model can be used for application to practical piping systems. Numerical examples are presented which do not predict significant shear and bending, and hence pose no problem numerically, but otherwise demonstrate relevant structural motion along with transient pressures and velocities in the contained liquid. Experimental data is used to partially verify the wave model.

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References

1 Wiggert, D. C., Otwell, R. S., and Hatfield, F. J., "The Effect of Elbow Restraints on Pressure Transients," ASME JOURNAL OF FLUIDS ENGINEERING, Vol. 107, No. 3, Sept. 1985.

2 Skalak, R., "An Extension of the Theory of Water Hammer," *Trans. ASME*, Vol. 78, No. 1, 1956.

3 Thorley, A. R. D., "Pressure Transients in Hydraulic Pipelines," ASME *Journal of Basic Engineering*, Vol. 91, Sept. 1969.

4 Williams, D. J., "Waterhammer in Non-Rigid Pipes: Precursor Waves and Mechanical Damping," *Journal of Mechanical Engineering Science*, Vol. 19, No. 6, Institute of Mechanical Engineers, 1977.

5 Walker, J. S., and Phillips, J. W., "Pulse Propagation in Fluid Filled Tubes," ASME *Journal of Applied Mechanics*, Mar. 1977.

6 Wilkinson, D. H., "Acoustic and Mechanical Vibrations in Liquid Filled Pipework Systems," *Proceedings of the Vibration in Nuclear Plant Conference*, British Nuclear Engineering Society, May 1978.

7 Wilkinson, D. H., and Curtis, E. M., "Water Hammer in a Thin-Walled Pipe," *Proceedings 3rd International Conference on Pressure Surges*, Vol. 1, British Hydromechanics Research Association, Mar. 1980.

8 Ellis, J., "A Study of Pipe-Liquid Interaction Following Pump Trip and Check-Valve Closure in a Pumping Station," *Proceedings 3rd International Conference on Pressure Surges*, Vol. 1, British Hydromechanics Research Association, Mar. 1980.

9 Timoshenko, S., and Young, D. H., *Vibration Problems in Engineering*, 3rd ed., D. Van Nostrand Company, New York.

10 Forsythe, G. E., and Wasow, W. R., *Finite-Difference Methods for Partial Differential Equations*, Wiley, New York, 1960.

11 Leonard, R. W., and Budiansky, B., "On Traveling Waves in Beams," National Advisory Committee for Aeronautics, Report 1173, 1955.

12 Goldberg, D. E., and Wylie, E. B., "Characteristics Method Using Time-Line Interpolations," *Journal of Hydraulic Engineering*, Vol. 109, No. 5, American Society of Civil Engineers, May, 1983.