Magnetic Force and Torque Calculation Between Circular Coils With Nonparallel Axes

Zhong Jian Wang and Yong Ren

***Abstract*—A method for calculating the magnetic force and torque between circular coils with nonparallel axes and rectangular cross section is presented. In the method, the magnetic field values are expressed in terms of the complete elliptical integrals of the first and second kind. The magnetic force and torque results are obtained by adding all magnetic forces and torques of the filaments. The method can apply to all circumstances between the two circular coils in a vacuum permeability environment. In addition, another comparative method based on mutual inductance gradient is given. The accuracy of the method is discussed in this paper. Results obtained by the two approaches are in excellent agreement.**

***Index Terms*—Circular coil, filament, magnetic force, nonparallel, torque.**

I. Introduction

The steady high magnetic field facility (SHMFF) is being constructed at the High Magnetic Field Laboratory, Chinese Academy of Sciences. The SHMFF system includes four water-cooled magnets, four superconducting magnets, and one 40-T hybrid magnet [1]. Each magnet consists of several coils. During the installation of these coils, the misalignment is inevitable, such as axial offset, radial offset, or nonparallel axes. With the effect of these misalignments, an extra interaction force and torque is produced, which will have an impact on the designs of the magnet support. Thus, an accurate evaluation of the force and torque is very important. There are three basic circumstances for the position relation of two circular coils [2]. The first is coaxial but has an axial offset. The second is coincident for the midplane between two coils but has a radial offset. The last is coincident for the center points between two coils but has an angle consisting of two axes. The position relation for two coils in space may be the combination of three. For a coaxial circumstance, the magnetic force calculation has been discussed in [2]–[6]. For two coils with parallel axes, the magnetic force calculation has been discussed in [7] and [8].

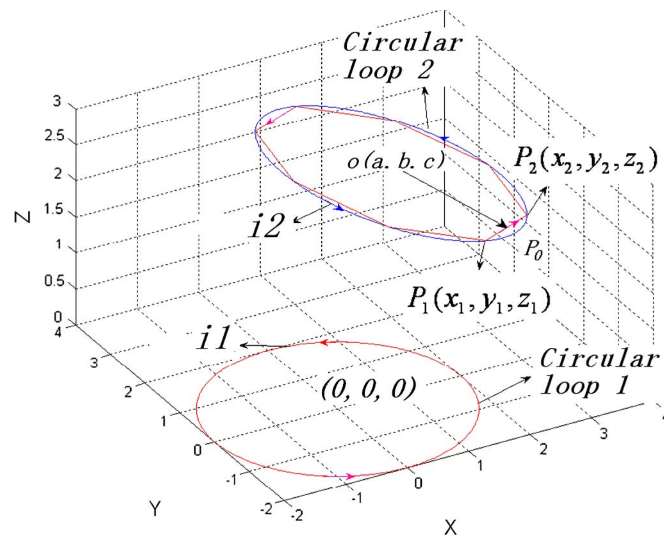

Fig. 1. Two coils with nonparallel axes.

In these two cases, the axes of two coils share one common plane. If the axes of two coils share one common plane, the mutual inductance for two coils can be calculated [9]–[11], [15], [16], and the magnetic force can be obtained by calculating the mutual inductance gradient [3], [8]. However, in space, the nonparallel axes of two coils are often seen. In the nonparallel axes, the mutual inductance calculation is difficult. Thus, the mutual inductance gradient method is also inconvenient. The method in this paper can solve this problem. The method consists of two parts. First, we get an expression of magnetic force and torque calculation for two circular current loops in arbitrary position in a vacuum permeability environment. Second, we calculate the magnetic force and torque results of a superconducting magnet with this method and compare them with that of the mutual inductance gradient method.

II. Magnetic Force Calculation for Two Circular Current Loops

There are two circular loops in Fig. 1. i1 stands for the current of circular loop 1, and i2 stands for the current of circular loop 2. For simplicity, circular loop 1 is in the XOY plane and the center at the original point. In order to calculate the magnetic force between the two circular loops, we must calculate the magnetic field produced by circular loop 1 on circular loop 2 with the following formulas (as shown in Fig. 2) [12]–[14].

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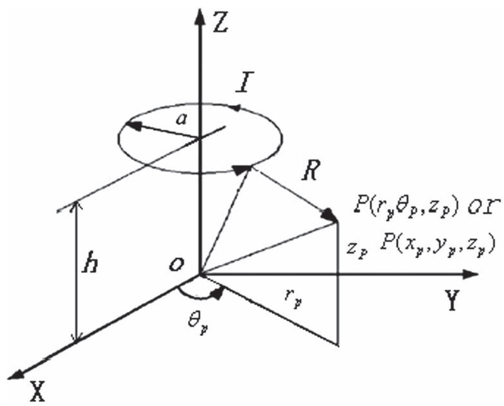

Fig. 2. Field point and a circular current loop.

$K(k)$: Complete elliptic integral of the first kind.
$E(k)$: Complete elliptic integral of the second kind.

$$B_r(r_p, z_p) = -\frac{\mu_0 I}{2\pi} \frac{z_p - h}{r_p\sqrt{(r_p+a)^2 + (z_p-h)^2}} \cdot \left\{ K(k) - \frac{r_p^2 + a^2 + (z_p-h)^2}{(r_p-a)^2 + (z_p-h)^2} E(k) \right\} \quad (1)$$

$$B_z(r_p, z_p) = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{(r_p+a)^2 + (z_p-h)^2}} \cdot \left\{ K(k) - \frac{r_p^2 - a^2 + (z_p-h)^2}{(r_p-a)^2 + (z_p-h)^2} E(k) \right\} \quad (2)$$

$$K(k, \phi_1 = 0, \phi_2 = \pi/2) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2 \alpha}} \quad (3)$$

$$E(k, \phi_1 = 0, \phi_2 = \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \alpha}\, d\alpha \quad (4)$$

$$\left\{4r_p a / \left[(r_p+a)^2 + (z_p-h)^2\right]\right\}^{1/2}. \quad (5)$$

In the Cartesian coordinate system, the magnetic field at point P (xp, yp, zp) can be expressed by the following formulas (as shown in Fig. 2):

$$Bx = Br \cdot \frac{x_p}{r_p} \quad (6)$$

$$By = Br \cdot \frac{y_p}{r_p} \quad (7)$$

$$Bz = Bz \quad (8)$$

$$r_p = \sqrt{x_p^2 + y_p^2}. \quad (9)$$

With formulas (1)–(9), we can easily get the magnetic field values at point $P1$ and point $P2$ in Fig. 1. For simplicity, $Bx1$ stands for the X-component of the magnetic field at point $P1$, $By1$ stands for the Y-component, and $Bz1$ stands for the Z-component. For the same expression, the three components of the magnetic field at point $P2$ are $Bx1$, $By1$, and $Bz1$, respectively. The average magnetic field along filament $\overrightarrow{P_1P_2}$ can be expressed by the following formulas:

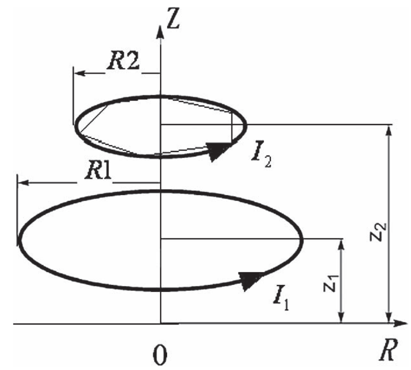

Fig. 3. Two coaxial circular loops.

$$Bx_{1.2} = \frac{Bx_1 + Bx_2}{2} \quad (10)$$

$$By_{1.2} = \frac{By_1 + By_2}{2} \quad (11)$$

$$Bz_{1.2} = \frac{Bz_1 + Bz_2}{2}. \quad (12)$$

The magnetic force of filament $\overrightarrow{P_1P_2}$ and torque at the center of circular loop 2 caused by circular loop 1 can be expressed by the following formulas:

$$\overrightarrow{dF_1^2} = i2 \cdot \overrightarrow{dl} \times \overrightarrow{B_{1.2}} \quad (13)$$

$$\overrightarrow{dl} = (x_2 - x_1) \cdot \bar{i} + (y_2 - y_1) \cdot \bar{j} + (z_2 - z_1) \cdot \bar{k} \quad (14)$$

$$\overrightarrow{B_{1.2}} = Bx_{1.2} \cdot \bar{i} + By_{1.2} \cdot \bar{j} + Bz_{1.2} \cdot \bar{k} \quad (15)$$

$$\overrightarrow{dM_1^2} = \overline{OP_0} \times \overline{dF_1^2}. \quad (16)$$

$P0$ is the middle point of filament $\overrightarrow{P_1P_2}$. We can divide circular loop 2 into n (number of filaments) straight filaments and calculate the magnetic force and torque of each filament. Then, we can get the whole magnetic force and torque of circular loop 2 caused by circular loop 1 by using the following formulas:

$$\overrightarrow{F_t} = \sum_{i=1}^{n} \overrightarrow{dF_i^{i+1}} \quad (17)$$

$$\overrightarrow{M_t} = \sum_{i=1}^{n} \overrightarrow{dM_i^{i+1}}. \quad (18)$$

The relationship between the n value and the accuracy of result can be seen through example I of magnetic force calculation.

Example I: Calculate the magnetic force between the two coaxial circular loops with the following dimensions (as shown in Fig. 3):

$$R1 = 0.5 \text{ m}, \quad Z1 = 0.2 \text{ m}, \quad I1 = 100 \text{ A}$$

$$R2 = 0.3 \text{ m}, \quad Z2 = 0.6 \text{ m}, \quad I2 = 200 \text{ A}.$$

With the mutual inductance gradient method [3], the magnetic force between the two circular current loops can be

TABLE I
MAGNETIC FORCE CALCULATION OF EXAMPLE I WITH THE METHOD IN THIS PAPER (COMPARISON WITH $Fz = -9.7911800$ mN)

The n value	The magnetic force Fz (mN)	Error (%)
8	-8.8151571	9.9684
16	-9.5414561	2.5505
32	-9.7283845	0.6413
64	-9.7754558	0.1606
100	-9.7847349	0.0658
128	-9.7872450	0.0402
256	-9.7901937	0.0101
512	-9.7909309	0.0025
1024	-9.7911153	0.0007
2048	-9.7911614	0.0002
3000	-9.7911696	0.0001
6000	-9.7911749	0.0001

TABLE II
DESIGN PARAMETER OF THE SUPERCONDUCTING MAGNET

Coil type	Nb3Sn CICC Coil	NbTi Solenoid Coils	
Coil number	Coil1	Coil 2	Coil3
Inner diameter(mm)	120.4	336.0	393.0
Outer diameter(mm)	266.0	393.0	480.3
Height(mm)	529.2	552.0	552.0
Turns	126	1890	3792
Operating Current(A)	16500	725	725

expressed by

$$F = I_1 I_2 \frac{\partial M}{\partial Zq}. \tag{19}$$

For two coaxial circular loops, the force expression can be expressed by the following formulas [4], i.e.,

$$F = Fz = \frac{\mu_0 I_1 I_2 k Zq}{4\sqrt{R_1 R_2}} \left[\frac{(2-k^2)}{1-k^2} E(k) - 2K(k) \right] \tag{20}$$

$$k^2 = \frac{4R_1 R_2}{(R_1+R_2)^2 + Zq^2} \tag{21}$$

$$Zq = Z_2 - Z_1. \tag{22}$$

With the mutual inductance gradient method, the magnetic force is $Fz = -9.7911800$ mN.

With the new filament method in this paper, we can get the magnetic force value in Table I.

From Table I, we can see that the magnetic force result is more and more accurate as the n value is becoming bigger and bigger. However, the time of calculation is longer and longer. In order to save calculation time and guarantee the accuracy of magnetic force result, we must choose a proper n value.

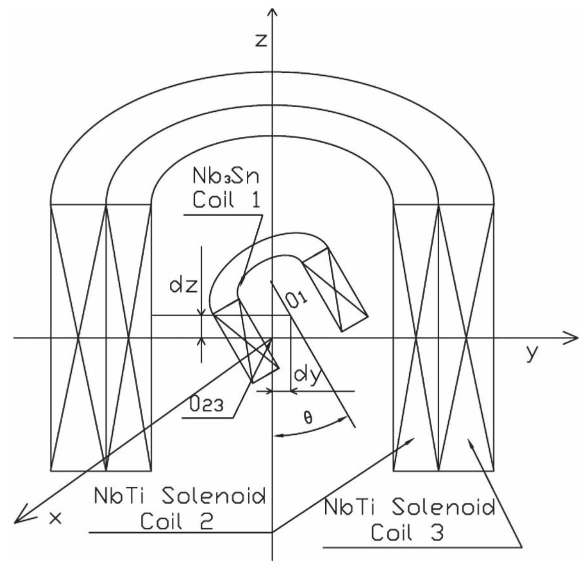

Fig. 4. Radial and axial misalignment for two types of coils.

From Table I, we can also see that $n = 256$ is enough for the engineering calculation. The magnetic force results obtained by the two methods are in very excellent agreement. The torque accuracy with n is the same as the result of magnetic force.

III. MAGNETIC FORCE AND TORQUE CALCULATION COMPARISON BETWEEN THIS WORK AND THE MUTUAL INDUCTANCE GRADIENT METHOD THROUGH A SUPERCONDUCTING MAGNET

The superconducting magnet has been finished at the High Magnetic Field Laboratory, Chinese Academy of Sciences. The magnet with one Nb3Sn cable-in-conduit conductor (CICC) coil and the two NbTi solenoid coils will provide a central magnetic field of 12 T. Table II lists the design parameters of the superconducting magnet. In order to calculate the magnetic force and torque between the Nb3Sn CICC coil and the two NbTi solenoid coils, we divide each coil into several cells and assume that the current density in the cross section for each coil is uniform. Each cell represents a filament circular loop [9], [14]. For the Nb3Sn CICC coil, each filament circular loop consists of $n = 256$ filaments.

Example II: Calculate the magnetic force and torque between two types of coils as Table II (position relation, as shown in Fig. 4).

O1 Center point of the Nb3Sn CICC coil.
O2 Common center point of the two NbTi solenoid coils.
dx Distance along x-axis between O1 and O23.
dy Distance along y-axis between O1 and O23.
dz Axial distance between O1 and O23.
θ Angle between axes in x-axis direction.

$$T = I_1 I_2 \frac{\partial M}{\partial \theta}. \tag{23}$$

From Tables III and IV, we can see that the magnetic force results obtained by the two approaches are in excellent

TABLE III
AXIAL MAGNETIC FORCE (N) CALCULATION OF THIS WORK [SUBLINE (1) OF EACH LINE] ($dx = 0; \theta = 0$). AXIAL MAGNETIC FORCE (N) CALCULATION OF THE MUTUAL INDUCTANCE GRADIENT METHOD [SUBLINE (2) OF EACH LINE] ($dx = 0; \theta = 0$) [8]. ERROR BETWEEN THE TWO METHODS [SUBLINE (3) OF EACH LINE]

dz \ dy	0mm	2mm	4mm
0 mm	0	0	0
	0	0	0
1 mm	6011	6012	6015
	6035	6036	6038
	0.4%	0.4%	0.38%
2 mm	12021	12023	12028
	12040	12041	12046
	0.16%	0.15%	0.15%
3 mm	18031	18033	18040
	18043	18045	18052
	0.067%	0.067%	0.067%
4 mm	24037	24040	24050
	24043	24047	24056
	0.025%	0.029%	0.025%

TABLE IV
RADIAL MAGNETIC FORCE (N) CALCULATION OF THIS WORK [SUBLINE (1) OF EACH LINE] ($dx = 0; \theta = 0$). RADIAL MAGNETIC FORCE (N) CALCULATION OF THE MUTUAL INDUCTANCE GRADIENT METHOD [SUBLINE (2) OF EACH LINE] ($dx = 0; \ \theta = 0$) [8]. ERROR BETWEEN THE TWO METHODS [SUBLINE (3) OF EACH LINE]

dz \ dy	0mm	2mm	4mm
0 mm	0	6012	12026
	0	6019	12027
		0.12%	0.01%
1 mm	0	6011	12025
	0	6018	12026
		0.12%	0.01%
2 mm	0	6010	12023
	0	6017	12024
		0.12%	0.01%
3 mm	0	6008	12019
	0	6015	12020
		0.12%	0.01%
4 mm	0	6005	12013
	0	6012	12014
		0.12%	0.01%

agreement. The torque results in Table V are also in excellent agreement.

Table VI lists several magnetic force and torque results between the Nb3Sn CICC coil and the two NbTi solenoid coils with nonparallel axes in a vacuum permeability environment.

IV. CONCLUSION

An accurate filament method for magnetic force and torque calculation between circular coils has been presented in this paper. The method applies to the magnetic force and torque calculation between circular coils with parallel and non-parallel axes. The validity of the method has been verified by the mutual inductance gradient method. The accuracy with the method can be controlled by adjusting the value of n and the density of cells.

TABLE V
TORQUE CALCULATION $\overrightarrow{M_1}$ OF THIS WORK ($dx = 0; dy = 0; dz = 0$). TORQUE CALCULATION $\overrightarrow{M_2}$ OF THE MUTUAL INDUCTANCE GRADIENT METHOD WITH FORMULA (23)

Angle θ (°)	$\overrightarrow{M}_1$ (Nm)	$\overrightarrow{M}_2$(Nm)	Error(%)
0	0	0	0
0.5	1068.1	1078.0	0.92
1.0	2138.3	2147.3	0.42
1.5	3212.4	3220.6	0.25
2.0	4292.6	4299.9	0.17
2.5	5380.9	5387.3	0.12
3.0	6479.5	6484.8	0.082
3.5	7590.5	7594.5	0.053
4.0	8716.1	8718.9	0.032
4.5	9858.6	9859.9	0.013
5.0	11020.3	11020.0	0.003

TABLE VI
MAGNETIC FORCE AND TORQUE RESULTS BETWEEN THE Nb3Sn CICC COIL AND THE TWO NbTi SOLENOID COILS WITH NONPARALLEL AXES IN A VACUUM PERMEABILITY ENVIRONMENT

Coils relative position	$\overrightarrow{F}$ (N)	$\overrightarrow{M_2}$ (Nm)
dx=1mm dy=0mm dz=0mm θ=1°	Fx= 3006.2 Fy= 0 Fz= 0	Mx=-2138.2 My= 0 Mz= 0
dx=1mm dy=0mm dz=1mm θ=1°	Fx= 3006.0 Fy=-43.00 Fz=-6013.8	Mx=-2138.4 My= 2.4630 Mz= 0.0430
dx=1mm dy=1mm dz=1mm θ=1°	Fx= 3006.1 Fy= 2964.6 Fz=-6057.1	Mx=-2140.8 My= 2.3735 Mz= 0.0414

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