Magnetic Force Calculation Between Misaligned Coils for a Superconducting Magnet

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***Abstract*—A superconducting magnet system that consists of a Nb_3Sn cable-in-conduit conductor (CICC) coil and two NbTi solenoid coils is being developed. The superconducting magnet will be capable of generating a central magnetic field of 12 T. The stored energy of the magnet is 2.05 MJ. As a result of possible misalignment between the Nb_3Sn coil and NbTi coils during installation, there exists a radial or axial force between two types of coils. In this paper, we use the Grover's formulas with filament method to calculate the mutual inductance and its gradient between coils. Based on this method, we apply the mutual inductance gradient method to calculate the radial and axial force between coils. We confirm the validity of this method by comparing it with an exact 3-D semianalytical expression on axial force for coaxial but midplane offset coils. Results obtained by two methods are in excellent agreement. All results obtained on radial and axial forces are introduced.**

***Index Terms*—Cable-in-conduit conductor (CICC), magnetic force, misalignment, mutual inductance, superconducting magnet.**

I. Introduction

A superconducting magnet is being constructed, aimed at testing the performance of a cable-in-conduit conductor (CICC) of a model coil for 40-T hybrid magnet superconducting outsert at the High Magnetic Field Laboratory, Chinese Academy of Sciences. The magnet, which consists of a Nb_3Sn CICC coil and two NbTi solenoid coils, will provide a central magnetic field of 12 T in a cold bore of 120 mm [1]. The NbTi solenoid coils have been constructed as part of a 20-T hybrid magnet 20 years ago, which will be updated to an outsert magnet for the 12-T superconducting magnet [2]. As a result of possible misalignment of two types of coils during installation, such as axial offset or radial offset, an interaction force in the axial direction as a restoring force or an interaction force in the radial direction as a propulsion force between two types of coils will have an important impact on the design of the structural support. Therefore, an accurate evaluation of the interaction force is required. Many contributions to the interaction force calculation have been performed [3]–[10]. In this paper, we use the mutual inductance gradient method to calculate the radial and axial force. The electromagnetic force is proportional to the mutual inductance gradient with respect to axial offset of coil midplanes and radial offset of coil axes [6], [11]. For simplicity, we only calculate the interaction force for nontilted coils that are offset axially and radially, which is the worst case. The mutual inductance between two axial offset or radial offset filamentary circular coils with parallel axes can be calculated by the well-known Grover's formulas [10], [11]. On the basis of the Grover's formulas, we apply the Simpson's numerical integration to calculate the mutual inductance between coils of the superconducting magnet. In order to evaluate the accuracy of the method, we use an exact 3-D semianalytical expression to calculate the axial force between coaxial coils as a comparative method. The accuracy meets the demand for designing. In this paper, the mutual inductance and the interaction force between coils are presented.

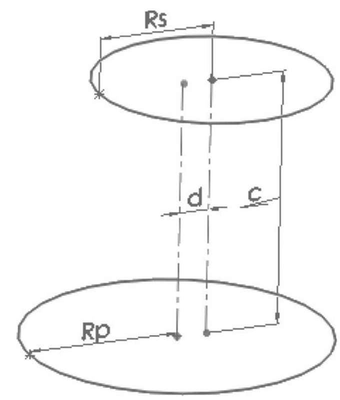

Fig. 1. Filamentary circular coils with misalignment.

II. Electromagnetic Force Expressions

The electromagnetic force between two current carrying coils with radial or axial misalignment can be derived from the general expression with their mutual inductance gradient [6], [11]

$$F = \frac{\partial M_{1,2}}{\partial d} I_1 I_2 \tag{1}$$

where I_1 and I_2 are the currents of two coils, $M_{1,2}$ is their mutual inductance, and d is the generalized coordinate. The mutual inductance between two axial offset or radial offset filamentary circular coils with parallel axes, as shown in Fig. 1, can be expressed as [11]

$$M_{1,2} = \frac{\mu_0}{\pi}\sqrt{R_P R_S}\int_0^{\pi} \frac{\left(1 - \frac{d}{R_S}\cos\vartheta\right)\Phi(k)}{\sqrt{v^3}} d\vartheta \tag{2}$$

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TABLE I
DESIGN PARAMETERS OF A SUPERCONDUCTING MAGNET

Coil name	Nb_3Sn CICC	NbTi solenoid coils	
	Coil 1	Coil 2	Coil 3
Inner diameter (mm)	120.4	336.0	393.0
Outer diameter (mm)	266.0	393.0	480.3
Height (mm)	529.2	552.0	552.0
Turns	126	1890	3792
Operating current (A)	16500	725	
Stored energy (MJ)		2.05	
Central field (T)		12.14	

where

$$\alpha = \frac{R_S}{R_P} \quad \beta = \frac{c}{R_P} \quad k^2 = \frac{4\alpha v}{(1+\alpha v)^2 + \beta^2}$$

$$v = \sqrt{1 + \frac{d^2}{R_S^2} - 2\frac{d}{R_S}\cos\vartheta}$$

$$\Phi(k) = \left(\frac{2}{k} - k\right)K(k) - \frac{2}{k}E(k).$$

$K(k)$ complete elliptic integral of the first kind;
$E(k)$ complete elliptic integral of the second kind;
R_P radius of the primary coil;
R_S radius of the secondary coil;
c axial distance between coil midplanes;
d radial distance between axes.

III. NUMERICAL RESULTS

A. Magnet Design Parameters

The superconducting magnet consists of two parts: an inner part with a Nb_3Sn CICC coil and an outer part with two NbTi solenoid coils. The superconducting magnet would generate a central magnetic field of 12 T. The stored energy of the magnet is 2.05 MJ. Table I lists the design parameters of the superconducting magnet.

B. Calculation Method

In this paper, we use the filament method to calculate the mutual inductance between two circular coils of rectangular cross section with parallel axes. We divide the two types of coils into several cells. Each cell represents one filament. We assumed that the current density distribution is uniform in the cross section for each coil. We adopted Simpson's numerical integration method to calculate the mutual inductance of the two types of coils. Fig. 2 shows the mutual inductance gradient as a function of axial distance between coil midplanes for $\Delta d = 0.0,\ 2.5,$ and 5.0 mm, respectively. Fig. 3 shows the mutual inductance gradient as a function of radial distance between coil axes for $\Delta c = 0.0,\ 2.5,$ and 5.0 mm, respectively. The terms Δd and Δc are denoted as radial distance between coil axes and axial distance between coil midplanes. The approximation of a linear dependence between the mutual inductance gradient as a function of axial distance between coil midplanes and radial distance between coil axes is shown. Table II gives a selection of values of the axial forces between Nb_3Sn coil and NbTi coils for $\Delta d = 0.0,\ 1.0,\ 2.0,\ 3.0,\ 4.0,$ and 5.0 mm. Table III lists a selection of values of the radial forces between two misaligned coils for $\Delta c = 0.0,\ 1.0,\ 2.0,\ 3.0,\ 4.0,$ and 5.0 mm. The results show that the axial force is restoring force, and the radial force is propulsion force. All results were performed in FORTRAN programming.

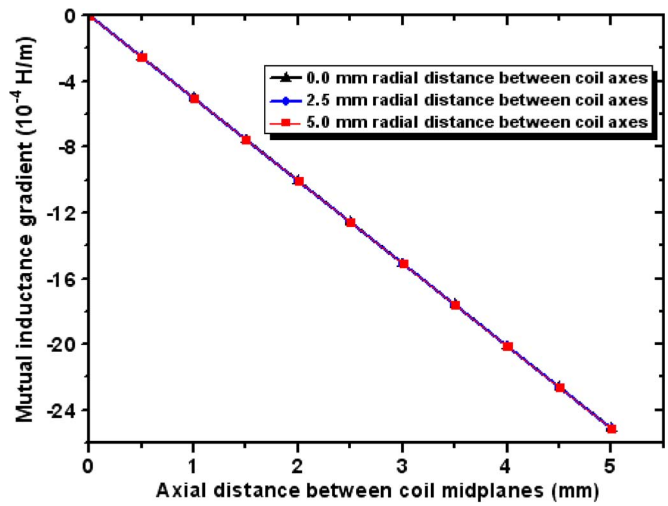

Fig. 2. Mutual inductance gradient as a function of axial distance between coil midplanes.

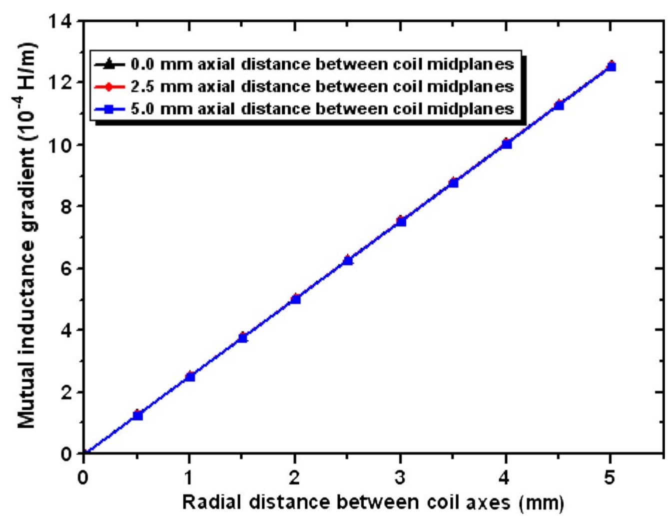

Fig. 3. Mutual inductance gradient as a function of radial distance between coil axes.

IV. COMPARISON WITH COAXIAL CASES FOR AXIAL FORCE CALCULATION

In order to evaluate the accuracy of the method for interaction force calculation, an exact 3-D semianalytical expression of axial force was adopted but only for coaxial coils. For two thick solenoid coils with coaxial, the axial force can be calculated by [5]

$$F_z = \frac{\mu_0 j_1 j_2}{2} \int_{r_1}^{r_2} \int_{0}^{2\pi} \sum_{i=1}^{2} (-1)^i \sum_{j,k=3}^{4} (-1)^{j+k+1} \cdot \left[f + gr^2 \cos\theta\right] \cos\theta d\theta dr \quad (3)$$

TABLE II
AXIAL FORCE BETWEEN MISALIGNED COILS (A NB_3SN COIL AND TWO NbTi COILS) FOR A SUPERCONDUCTING MAGNET

Δc (mm)	Δd (mm)	0.0	1.0	2.0	3.0	4.0	5.0
0.0	Force (N)	0	0	0	0	0	0
0.5	Force (N)	-3032	-3033	-3033	-3033	-3034	-3035
1.0	Force (N)	-6035	-6035	-6036	-6037	-6038	-6040
1.5	Force (N)	-9038	-9038	-9039	-9040	-9042	-9045
2.0	Force (N)	-12040	-12040	-12041	-12043	-12046	-12050
2.5	Force (N)	-15041	-15042	-15043	-15046	-15050	-15054
3.0	Force (N)	-18043	-18043	-18045	-18048	-18052	-18058
3.5	Force (N)	-21043	-21044	-21046	-21050	-21055	-21061
4.0	Force (N)	-24043	-24044	-24047	-24051	-24056	-24064
4.5	Force (N)	-27042	-27043	-27046	-27051	-27057	-27065
5.0	Force (N)	-30041	-30042	-30045	-30050	-30057	-30066

Note that Δc and Δd are denoted as axial distance between coil planes and radial distance between coil axes.

TABLE III
RADIAL FORCE BETWEEN MISALIGNED COILS (A NB_3SN COIL AND TWO NbTi COILS) FOR A SUPERCONDUCTING MAGNET

Δd (mm)	Δc (mm)	0.0	1.0	2.0	3.0	4.0	5.0
0.0	Force (N)	0	0	0	0	0	0
0.5	Force (N)	1515	1515	1515	1514	1514	1513
1.0	Force (N)	3016	3016	3016	3015	3013	3011
1.5	Force (N)	4518	4517	4516	4515	4513	4510
2.0	Force (N)	6019	6018	6017	6015	6012	6009
2.5	Force (N)	7520	7520	7518	7516	7512	7508
3.0	Force (N)	9022	9022	9020	9017	9012	9007
3.5	Force (N)	10524	10524	10522	10518	10513	10507
4.0	Force (N)	12027	12026	12024	12020	12014	12007
4.5	Force (N)	13530	13529	13526	13522	13515	13507
5.0	Force (N)	15033	15032	15029	15024	15017	15008

Note that Δc and Δd are denoted as axial distance between coil planes and radial distance between coil axes.

where

$$f = \frac{\alpha\varsigma r}{2} + \frac{\beta r}{2}\ln|\alpha+\varsigma|$$

$$g = -\varsigma + \gamma\arctan\left(\frac{\varsigma}{\gamma}\right) - \gamma\arctan\left(\frac{\varsigma\varepsilon}{\alpha\gamma}\right) + \varsigma\ln|\varepsilon+\alpha| + \varepsilon\ln|\alpha+\varsigma|$$

$$\alpha = \sqrt{r^2 + r_k^2 - 2rr_k\cos\theta + \varsigma^2}$$

$$\beta = r^2 + r_k^2 - 2rr_k\cos\theta$$

$$\gamma = r\sin\theta \quad \varepsilon = r_k - r\cos\theta \quad \varsigma = z_i - z_j$$

$r_k(k=1,2)$ inner or outer radius of the outer coil;
$r_k(k=3,4)$ inner or outer radius of the inner coil;
$z_k(k=1,2)$ lower or upper height of the outer coil;
$z_k(k=3,4)$ lower or upper height of the inner coil.

Table IV lists the values of the axial magnetic force obtained using the exact 3-D semianalytical expressions of the axial force [5]. The corresponding computational values, compared with the values calculated from (3), are also given. It can be noted that all values in (1) and (2) are in excellent agreement with those values obtained using (3). More precise results can be obtained if we increase the number of the meshes with the mutual inductance gradient method, which will take an enormous time to calculate the results. From the viewpoint of engineering, the accuracy of the results satisfies the demand in engineering. Thus, it is not necessary to take an enormous time to obtain a more precise result. The comparative calculation was also made in FORTRAN programming.

TABLE IV
COMPARISON BETWEEN OUR ANALYTICAL APPROACH, THE MUTUAL INDUCTANCE GRADIENT METHOD, AND AN EXACT 3-D SEMIANALYTICAL EXPRESSION FOR CALCULATING THE RADIAL FORCE BETWEEN TWO MISALIGNED COILS (A NB_3SN COIL AND TWO NbTi COILS) FOR A SUPERCONDUCTING MAGNET

Δc (mm)	Axial force (N) This work	Axial force (N) [12]	Error
0.0	0	0	0.00%
0.5	-3001	-3032	1.03%
1.0	-6003	-6035	0.53%
1.5	-9005	-9038	0.36%
2.0	-12000	-12040	0.28%
2.5	-15007	-15041	0.23%
3.0	-18008	-18043	0.19%
3.5	-21008	-21043	0.16%
4.0	-24007	-24042	0.15%
4.5	-27005	-27042	0.14%
5.0	-30003	-30041	0.13%

Note that Δc is denoted as axial distance between coil planes.

V. CONCLUSION

The mutual inductance and the interaction force between misaligned coils for a superconducting magnet have been calculated in this paper. The results have been calculated using FORTRAN programming. We have assumed that the current density distribution is uniform in a cross section for each coil. The calculation of mutual inductance has been obtained from Grover's formula using Simpson's numerical integration method. The interaction force has been evaluated by mutual inductance gradient method based on the results from the obtained mutual inductance. The formula has been validated by comparison with other numerical integration formulas for axial force calculation between coaxial coils. Results obtained by both methods are in excellent agreement with each other for axial magnetic force with coaxial coils. The calculation results have shown that the radial force between coils is propulsion force and the axial force is restoring force. Based on the results, a support structure for withstanding the interaction forces for the superconducting magnet will be designed.

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