# Dynamic Analysis of Fluid-Structure Interaction for the Biped Robot Running on Water 

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#### Abstract

A novel biped robot inspired by basilisk lizards is designed to simulate the water-running function. The kinematic analysis of the running-mechanism is brought out to get the movement equations of the mechanism, and the numerical simulation results show that the feet trajectories of the robot are similar to basilisk lizards. For calculating the pressure distribution on a robot foot before touching water surface, the compression flow of air and depression motion of water surface are considered. The calculating model after touching water surface has been built according to the theory of the plane motion of a rigid body. The multi-material ALE algorithm is applied to emulate the course of the foot slapping water. Numerical simulation results indicate that the model of the bionic robot can satisfy the water-running function.


Keywords-Biped Robot, Running on Water, Dynamic Analysis, Fluid-Structure Interaction

## I. INTRODUCTION

When a basilisk lizard is running on water surface, its feet slap water surface to pressure water to go down or spread, then an air cave comes into being above and around them, which produces the lift force overcome their weight and the forward thrust for water running. Compared with the moving method of a steamboat, this method can reduce the viscous drag force of water, which may improve the driver efficiency greatly. A quadruped robot was designed to imitate the water-running function, and the parameters of the robot, such as the feet size and the legs length, were calculated and optimized [1]-[3]. The robot can service on martial reconnaissance, monitoring water quality, exploring marsh and rescuing lives in floods, and so on [4].

A novel biped robot has been carried out to simulate the water-running function of basilisk lizards, whose aim is to develop an amphibian biped robot which can walk on land
and water surface [5], but the impact pressure of water to the robot was not discussed. When the robot feet impact water surface, water surface will produce the reaction force to the feet, so the fluid-structure interaction between the feet and the fluid must be considered to obtain the lifting force of the biped robot.

In this paper, the kinematics analysis is brought out to obtain the parameters of the biped water-running mechanism, and the velocity and acceleration are known. The fluid-structure interaction dynamic analysis is carried out to get the reaction pressure of water surface. A fluid-structure interaction theory model is built by taking account into the compression flow of air and depression motion of water surface in cylindrical coordinates, and the deformation of the foot is ignored.

## II. Kinematic analysis of the biped robot

Reference [5] shows a virtual prototype of the biped robot running on water surface, which is shown as Fig.1. For imitating the trajectory and function of the lizard foot, the mechanism of the robot leg is composed of an under-actuated six-bar linkage.


Fig. 1 Virtual Prototype of Biped Robot Running on Water
Fig. 2 shows the mechanism model of the biped robot leg, whose relatively fixed points are supposed on the robot body.


Fig. 2 Mechanism Model of Biped Robot Legs
The kinematic analysis of the above mechanism is carried out by using complex number vector method. The mechanism is divided into four linkage groups shown in Fig.3, and symbols of their parameters are shown in the figures.

In Fig.3, $l_{i}(i=1,4,5), k_{j}(j=2,3)$ represent the lengths of every linkage respectively, and $s_{2}, s_{3}$ represent the distances of $|B E|$ and $|A C|$ respectively. Supposed that $\varphi_{m}, \omega_{m}$ and $\alpha_{m}$ ( $m=1, \cdots, 5$ ) represent the position angles, angular velocities and angular accelerations of every linkage respectively.

The driving angular velocity of the mechanism is a constant $\left(\omega_{1}\right)$, the trajectory of Point $E$ is as follows

$$
\left\{\begin{array}{l}
x_{E}=l_{1} \cos \varphi_{1}  \tag{1}\\
y_{E}=l_{1} \sin \varphi_{1}
\end{array}\right.
$$

And the velocity and the acceleration can be deduced from the above formulas, which are respectively as follows


Fig. 3 Linkage Groups of Water-Running Mechanism

$$
\begin{align*}
& \left\{\begin{array}{l}
v_{E x}=-\omega_{1} l_{1} \sin \varphi_{1} \\
v_{E y}=-\omega_{1} l_{1} \cos \varphi_{1}
\end{array}\right.  \tag{2}\\
& \left\{\begin{array}{l}
a_{E x}=-\omega_{1}^{2} l_{1} \cos \varphi_{1} \\
a_{E y}=-\omega_{1}^{2} l_{1} \sin \varphi_{1}
\end{array}\right. \tag{3}
\end{align*}
$$

It can be get that $\theta_{3}=0$ from Fig.2, and the trajectories of Point $A\left(x_{A}, y_{A}\right)$ and Point $B\left(x_{B}, y_{B}\right)$ are

$$
\begin{align*}
& \left\{\begin{array}{l}
x_{A}=x_{E}+k_{2} \cos \varphi_{2} \\
y_{A}=y_{E}+k_{2} \sin \varphi_{2}
\end{array}\right.  \tag{4}\\
& \left\{\begin{array}{l}
x_{B}=x_{E}-s_{2} \cos \varphi_{2} \\
y_{B}=y_{E}-s_{2} \sin \varphi_{2}
\end{array}\right. \tag{5}
\end{align*}
$$

Similarly, $\theta_{3}=0$, and the trajectories of Point $O_{2}\left(x_{2}, y_{2}\right)$ and Point $C\left(x_{C}, y_{C}\right)$ are

$$
\left\{\begin{array}{l}
x_{2}=x_{A}+k_{3} \cos \varphi_{3}  \tag{6}\\
y_{2}=y_{A}+k_{3} \sin \varphi_{3}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
x_{C}=x_{A}+s_{3} \cos \varphi_{3}  \tag{7}\\
y_{C}=y_{A}+s_{3} \sin \varphi_{3}
\end{array}\right.
$$

The values of the eight parameters $\left(x_{A}, y_{A}, x_{B}, y_{B}, x_{C}, y_{C}, \varphi_{2}\right.$, $\varphi_{3}$ ) can be deduced from the above four equations, and by which the velocity $\left(v_{B x}, v_{B y}\right)$ and acceleration of Point $B$ can be known that

$$
\begin{gather*}
\left\{\begin{array}{l}
v_{B x}=v_{E x}+\omega_{2} s_{2} \sin \varphi_{2} \\
v_{B y}=v_{E y}-\omega_{2} s_{2} \cos \varphi_{2}
\end{array}\right.  \tag{8}\\
\left\{\begin{array}{l}
a_{B x}=a_{E x}+\alpha_{2} s_{2} \sin \varphi_{2}-\omega_{2}^{2} s_{2} \cos \varphi_{2} \\
a_{B y}=a_{E y}+\alpha_{2} s_{2} \cos \varphi_{2}+\omega_{2}^{2} s_{2} \sin \varphi_{2}
\end{array}\right. \tag{9}
\end{gather*}
$$

Eq.(5), (8) and (9) are the movement equations of Point $B$, which imitates the movement of the ankle of lizards.

Suppose that $\omega_{1}=80 \mathrm{r} / \mathrm{min}, l_{1}=20 \mathrm{~mm}, s_{2}=45 \mathrm{~mm}, k_{2}=35 \mathrm{~mm}$, $s_{3}=24 \mathrm{~mm}, k_{3}=100 \mathrm{~mm}, l_{4}=115 \mathrm{~mm}$ and $l_{5}=55 \mathrm{~mm}$, the curve of the trajectory, velocities and accelerations of Point $B$ can be got from the above equations, which are shown as Fig.4, Fig. 5 and Fig. 6.

According to the above curves, it can be known that the trajectory of Point $B$ is similar to the ankle of the lizard, which proves the mechanism can be used to imitate the function of water-running.


Fig 4 Trajectory of Point $B$


Fig 5 Curve of $v_{B}$


Fig. 6 Curve of $a_{B}$
The trajectory of Point $D$ is

$$
\left\{\begin{array}{l}
x_{D}=x_{C}+l_{4} \cos \varphi_{4}  \tag{10}\\
y_{D}=y_{C}+l_{4} \sin \varphi_{4}
\end{array}\right.
$$

so the angular velocity of Linkage $B D$ is obtained as

$$
\begin{equation*}
\omega_{5}=\frac{\left(v_{B x}-v_{C x}\right)\left(x_{D}-x_{C}\right)+\left(v_{B y}-v_{C y}\right)\left(y_{D}-y_{C}\right)}{\left(y_{D}-y_{B}\right)\left(x_{D}-x_{C}\right)-\left(y_{D}-y_{C}\right)\left(x_{D}-x_{B}\right)} . \tag{11}
\end{equation*}
$$

And the velocity of the centroid of Linkage $B D$ is

$$
\begin{equation*}
\boldsymbol{v}_{5}=\boldsymbol{v}_{B}+\omega_{5} \cdot \frac{l_{5}}{2} \tag{12}
\end{equation*}
$$

where $v_{B}^{2}=v_{B x}^{2}+v_{B y}^{2}$.

## III. Dynamic Analysis of Fluid-Structure Interaction of the robot

## A. Equations and Press Distribution of Air Layer before Touching Water

The flotage and driving force are produced in the stroke and slap phases during basilisk lizards running on water [5], and the air cave comes into being in the two phases. The fluid-structure interaction for the robot running on water should be discussed to display the pressure distribution of the robot feet. Suppose that the robot feet are rigid bodies, and slap water surface horizontally. The driving force and torque applied on Linkage $B D$ by the running mechanism are

$$
\left\{\begin{array}{l}
\boldsymbol{F}_{5}=m_{5} \boldsymbol{a}_{5}  \tag{13}\\
\boldsymbol{M}_{5}=J_{5} \boldsymbol{\alpha}_{5}
\end{array},\right.
$$

where $\boldsymbol{a}_{5}=d \boldsymbol{v}_{5} / d t, \boldsymbol{\alpha}_{5}=d \boldsymbol{\omega}_{5} / d t$, and $m_{5}, J_{5}$ are the mass and rotational inertia of the foot respectively.

For calculating the pressure distribution of a robot foot at any time, the compression flow of air and depression motion of water surface are considered and the deformation of the circular feet is ignored, at last every step length is solved by numerical method in a cylindrical coordinates.


Fig. 7 Calculating Model before Touching Water
A cylindrical coordinates is built as Fig.5, and the rotational angle of the foot can be ignored in the section of $h / R \ll 1$, so the air flow is looked as one dimensional movement. Suppose that the air velocity is $u_{a}(r, t)$, the height from foot to water surface is $h(r, t)$, the height of free water surface is $y^{\prime}=\eta(r, t)$, the air pressure is $P(r, t)$, the atmospheric pressure is $P_{0}$, the air density is $\rho(r, t)$, and the air density of 1 atm is $P_{0}$. The air viscosity is ignored, and the air is supposed as isentropic compression, that is, $P / P_{0}=\left(\rho / \rho_{0}\right)^{\gamma}$, so the movement equation and the continuity equation of air are

$$
\begin{align*}
& {\left[\frac{\partial}{\partial t}+\left(u_{a}+C_{a}\right) \frac{\partial}{\partial r}\right]\left(u_{a}+\frac{2 C_{a}}{\gamma-1}\right)=-C_{a}\left(\frac{1}{h} \frac{\partial h}{\partial t}+\frac{u_{a}}{h} \frac{\partial h}{\partial r}+\frac{u_{a}}{r}\right),} \\
& {\left[\frac{\partial}{\partial t}+\left(u_{a}-C_{a}\right) \frac{\partial}{\partial r}\right]\left(u_{a}-\frac{2 C_{a}}{\gamma-1}\right)=C_{a}\left(\frac{1}{h} \frac{\partial h}{\partial t}+\frac{u_{a}}{h} \frac{\partial h}{\partial r}+\frac{u_{a}}{r}\right),} \tag{14}
\end{align*}
$$

where $C_{a}^{2}=\mathrm{d} P / \mathrm{d} \rho$ is the local velocity of sound.
Suppose that $t=0$ when the foot is parallel to water surface, when $h(r, t)=h_{0}, v(t)=v_{0}$ and $y_{B}^{\prime}=y_{D}^{\prime}$, namely,

$$
\begin{equation*}
\left(k_{2}+s_{2}\right) \sin \varphi_{2}+s_{3} \sin \varphi_{3}+l_{4} \sin \varphi_{4}=0 \tag{16}
\end{equation*}
$$

So

$$
\begin{equation*}
h(r, t)=h_{0}-\eta(r, t)-\int_{0}^{t} V(t) \mathrm{d} t \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
V(\mathrm{t})=V_{0}+g t+\frac{1}{M_{5}}\left[\int_{0}^{t} d t \int_{0}^{R} 2 \pi R\left(P-P_{0}\right) d r+F_{5}\right] \tag{18}
\end{equation*}
$$

where

$$
\begin{aligned}
\eta(r, t)= & \lim _{y^{\prime} \rightarrow 0^{-}}\left(-\frac{1}{\rho_{l}}\right) \int_{0}^{r} d \tau^{\prime} \int_{0}^{c^{\prime}} d \tau \int_{0}^{R} d S \int_{0}^{\infty} m^{2} e^{m y^{\prime}} \cos \sqrt{g m} \\
& (t-\tau) \cdot S\left[P(S, \tau)-P_{0}\right] J_{0}(m r) J_{0}(S r) d m
\end{aligned}
$$

is the water wave equation. In this equation, $\rho_{l}$ is the density of water and $J_{0}$ is zero order Bessel function

Simultaneously the pressure distribution of the foot can be got as

$$
\begin{equation*}
P(r, t)=P_{0}+\rho_{0} \cdot \frac{R^{2}-r^{2}}{4 h^{2}}\left[\frac{3}{2}\left(\frac{d h}{d t}\right)^{2}-h \frac{d^{2} h}{d t^{2}}\right] . \tag{19}
\end{equation*}
$$

where $h(t)=h_{0}-\int_{0}^{t} V(t) d t$, so the initial conditions are set as that

$$
\begin{gather*}
\frac{V_{0}}{C_{0}} \ll \frac{h_{0}}{R} \ll 1, \quad \eta(r, 0)=0  \tag{20}\\
\frac{u(r, 0)}{C_{0}}=\frac{V_{0} r}{2 h_{0} C_{0}} \tag{21}
\end{gather*}
$$

and

$$
\begin{align*}
\frac{C_{a}(r, 0)}{C_{0}} & =\left(\frac{P(r, 0)}{P_{0}}\right)^{\frac{\gamma-1}{2 \gamma}}  \tag{22}\\
& =\left\{1+\gamma \frac{R^{2}-r^{2}}{4 h_{0}^{2}}\left[\frac{3}{2} \frac{V_{0}^{2}}{C_{0}^{2}}+\frac{h_{0}}{C_{0}^{2}}\left(\frac{d V}{d t}\right)_{t=0}\right]\right\}^{\frac{\gamma-1}{2 \gamma}}
\end{align*}
$$

when $t=0$ and $0 \leqslant r \leqslant R$.
Because the foot is not in a high speed, the flow speed of air at the edge of the foot is not over the sound speed, where $P(R, t)=P_{0}$. So

$$
\begin{equation*}
C_{a}(R, t)=C_{0}, \quad 0 \leq t \leq t^{\prime} \tag{23}
\end{equation*}
$$

where $t^{\prime}$ satisfies the equation $u\left(R, t^{\prime}\right)=C_{0}$.

## B. Basic Equations after Touching Water

Suppose that the edge of the foot will touch water when $t=t_{1}$, and

$$
\begin{equation*}
\eta\left(R, t_{1}\right)=h_{0}-\int_{0}^{1} V(t) d t . \tag{24}
\end{equation*}
$$



Fig. 8 Calculating Model after Touching Water
The calculating model after touching water surface is built as Fig. 8 by ignoring the air escaping from the bottom of the foot. According to the theory of the plane motion of a rigid body, the motion equations are as follows:

$$
\left\{\begin{array}{l}
m_{5} \frac{d v_{5 y^{\prime}}}{d t}=-\left[2 \pi \int_{0}^{R} P(r, t) r d r-\pi r^{2} P_{0}\right]+m_{5} g+F_{5} \sin \beta \\
m_{5} \frac{d v_{r}}{d t}
\end{array}=F_{5} \cos \beta \quad .\right.
$$

where $v_{5 y}$, is the $y^{\prime}$-component of the centroid velocity of the foot in the cylindrical coordinates, which can be got by the coordinate transformation of $y$-component of $v_{5}$, and $v_{r}$ is the $r$-component of the centroid velocity of the foot. So the pressure distribution of the foot is
$P(r, t)=\frac{1}{r} \frac{d}{d r}\left[\frac{P_{0} r^{2}}{2}+\frac{m_{5} g+F_{5} \sin \beta}{2 \pi}-\frac{m_{5}}{2 \pi} \frac{d v_{5 y^{\prime}}}{d t}\right]$.
And the fluid motion equations under the foot are

$$
\begin{equation*}
\frac{\partial\left(r W_{r}\right)}{\partial r}+\frac{\partial W_{\theta}}{\partial \theta}+r \frac{\partial W_{y^{\prime}}}{\partial y^{\prime}}=0, \tag{27}
\end{equation*}
$$

and

$$
\left\{\begin{array}{l}
\frac{\partial W_{r}}{\partial t}+\frac{W_{r} \partial W_{r}}{\partial r}+\frac{W_{\theta}}{r} \frac{\partial W_{r}}{\partial \theta}+W_{y^{\prime}} \frac{\partial W_{r}}{\partial y^{\prime}}-\frac{W_{\theta}^{2}}{r}=F_{r}-\frac{1}{\rho_{l}} \frac{\partial P}{\partial r} \\
\frac{\partial W_{\theta}}{\partial t}+\frac{W_{r} \partial W_{\theta}}{\partial r}+\frac{W_{\theta}}{r} \frac{\partial W_{\theta}}{\partial \theta}+W_{y^{\prime}} \frac{\partial W_{\theta}}{\partial y^{\prime}}+\frac{W_{r} W_{\theta}}{r}=-\frac{1}{\rho_{l}} \frac{\partial P}{\partial \theta}, \\
\frac{\partial W_{y^{\prime}}}{\partial t}+\frac{W_{r} \partial W_{y^{\prime}}}{\partial r}+\frac{W_{\theta}}{r} \frac{\partial W_{y^{\prime}}}{\partial \theta}+W_{y^{\prime}} \frac{\partial W_{y^{\prime}}}{\partial y^{\prime}}=F_{y^{\prime}}-\frac{1}{\rho_{l}} \frac{\partial P}{\partial y^{\prime}} \tag{28}
\end{array}\right.
$$

where
$F_{r}=\left[2 \pi \int_{0}^{R} P(r, t) r d r-\pi r^{2} P_{0}\right] \cos \delta$,
$F_{y^{\prime}}=\left[2 \pi \int_{0}^{R} P(r, t) r d r-\pi r^{2} P_{0}\right] \sin \delta$.

## C. Numerical Simulation and Prototype

The explicit dynamics analysis has been applied for the dynamic emulating of a foot slapping water, attempting to achieve more exact result. Because the two kinds of fluid media, water and air, are involved, the multi-material ALE algorithm has been taken to emulate the movement of the fluids, and the single-point Euler/ALE multi-material method has been applied as the unit algorithm. A multi-material unit
means the flow among various materials is permitted in the mesh generated by this kind of unit, so various materials may be included in a mesh, and the transport equations of various materials in the mesh should be solved in the algorithm. The foot is supposed as a rigid body, whose deformation has been ignored, and the Lagrange algorithm is applied for the foot structure dynamic emulating.

The contour map of the fluid density at certain time during running on water surface is shown in Fig.9.


Fig. 9 Contour Map of Fluid Density
In the figure, the upper part is full of air, and the lower part is full of water. The water surface fluctuation and the variety of the fluid density are shown in this figure.

The contour map of the pressure distribution of water is shown as Fig. 10.

The distribution of the pressure applied on water surface by the foot is shown as Fig. 11.

It can be seen that the maximum pressure applied on water surface by the foot is at the edge of the foot.


Fig. 10 Contour Map of Pressure


It can be got from the above analysis that during the robot running on water, the foot slaps water surface to pressurize
water to go down or spread around them. At the same time, an air cave comes into being above and around the foot. So the slap action produces the lift force overcome the robot weight and the forward thrust to achieve water walking, which is same with a basilisk lizard running on water ${ }^{[16]}$.

## IV. CONCLUSION

The next research will be focused on the influence on the pressure applied on water surface with the changing of the foot size and the velocity touching water surface, attempting to achieve the maximum pressure without increasing the weight of the robot greatly.

Because the robot runs on water with contrary phase variations of the legs, which will produce a periodic changing torque in the longitudinal direction of the robot body, the balance method will be researched and a balance equipment will be designed. The water-running action is rhythmic, so a CPG control model will be built to control the legs and the balance equipment.

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