dSPACE-based PID Controller for A Linear Motor Driven Inverted Pendulum

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Abstract — *Inverted pendulum is a kind of typical platform for control theory verification. Noted as a non-linear, strong-coupling and natural instable system, it has been widely concerned for a long time. Permanent magnet linear synchronous motor (PMLSM) driven inverted pendulum (dDIP) [1] is a new member of present similar devices. With dDIP, various unexpected disturbances such as lag effect of a belt attached to a cart and errors caused by a rotary encoder while detecting the position of a cart can be eliminated or reduced to a small range.*

The real performance of the dDIP is researched in this paper. Based on dSPACE real time control system, the results showed that the controllability of the dDIP is fine, and it also proved that the PID strategy can be used for this typical non-linear system.

Index Terms — dDIP, PID control, direct drive, linear motor

I. INTRODUCTIONS

Inverted Pendulum, noted as a non-linear, strong-coupling and natural instable system, has been widely concerned for a long time. Not only used as a teaching instrument, it is also studied in the spheres of theory and technology which are usually connected with precision instruments, robot control [2], human motion mechanisms [3] and advanced gravitational wave detectors. Although the inverted pendulum has been with us for almost several decades, it is still worth of studying.

The linear motor driven inverted pendulum proposed in this paper, is a new member of the similar devices. The conventional inverted pendulums, with their moving parts are usually driven by mechanisms of pulley-belt or leadscrew connected to a rotary servo motor, are commonly influenced by adverse factors caused by mechanisms mentioned above. Pulley-belt brings lag effect while leadcrew produces backlash when it changes its direction. Instead of using the ordinary driving methods, we directly drove an inverted pendulum with a linear motor, which could effectively reduce or eliminate those disadvantages, and we named it as dDIP (direct driven inverted pendulum).

Dozens of strategies can successfully keep a pendulum staying inverted, for instances, the well-known LQ, fuzzy control [4] and neural network strategies which are commonly appeared in a variety of journals. In this paper, We adopted the conventional PID (Proportional-Integral-Differential) control as our testing measure. It is known that PID controller is one of the most widespread controllers in industrial applications [5], [6], and its tuning procedure is easy. Inverted pendulum, noted as a typical non-linear system, can be analyzed and controlled by state-space functions with a non-linear controller. In this paper, our goal is to find out the performance and characteristics of dDIP, especially the differences between a conventional one and ours. Because of the convenience of PID control, we adopted it as our first attempt. The PID controller in the paper is able to be transplanted to different systems.

II. PHYSICAL ANALYSIS AND MODELLING

A. Preparation of Modelling

The dDIP mainly consists of a pendulum, a rotary encoder, a linear motor (a linear encoder is included), a servo driver, a controller (dSPACE) and the related accessories. The pendulum is attached to the rotary encoder by a short shaft, which is fixed in a rolling bearing sustained by its pedestal. The bearing pedestal is fixed on the motor table, which can move to and fro alongside a linear guide.

The complex physical model of dDIP was simplified deliberately at first (in fact, the model is not necessary in a typical PID control), for the sake of simulating procedure.

An inverted pendulum can be usually seen from two different points of view. One can consider the inverted pendulum as a "single input, double outputs" system. Its input is a force applied by the motor while its outputs are the movement of the motor table along with the change of the pendulum angle. Also, one can see it as a "single input, single output" system and regards the pendulum angle variation as a disturbance adding to the system instead of a system output. Both aspects are able to work out the problem essentially, and we take the former one as our research routine.

One can regard an inverted pendulum as a cart-pendulum system, as air resistance and other disturbances are ignored.

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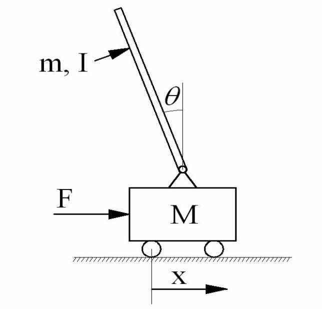

Fig. 1. The physical model of an inverted pendulum, simplified as a cart-pendulum system.

Fig. 1 shows a simplified cart-pendulum system. As the cart is bumped by an instantaneous force, the pendulum rotates an angle from the upright. The angle is such small that we can linearize the model around its equilibrium point, and then figure out its kinetic equations. We assume the following set of variables.

M	cart mass (kg)
m	pendulum quality (kg)
b	friction-velocity coefficient (Ns/m)
l	distance from the rotation point to the centroid of pendulum (m)
I	pendulum moment of inertia (kgm^2)
F	force exerts on cart (N)
x	cart position (m)
φ	angle between pendulum and the upper equilibrium point (rad)
θ	angle between pendulum and the lower equilibrium point (the initial state of pendulum is stay in its lower equilibrium point) (rad)

The following equations are cart-pendulum system's kinematic equations.

$$\begin{cases}(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F\\(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta\end{cases} \tag{1}$$

When the pendulum is around its equilibrium point, its angular variation is tiny, thus the former equations can be simplified as follows. Substitute u for F in the equations.

$$\begin{cases}(I+ml^2)\ddot{\phi}-mgl\phi=ml\ddot{x}\\(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u\end{cases} \tag{2}$$

B. The transfer functions

We can get the following equations after making Laplace transformation.

$$\begin{cases}(I+ml^2)\Phi(s)s^2-mgl\Phi(s)=mlX(s)s^2\\(M+m)X(s)s^2+bX(s)s-ml\Phi(s)s^2=U(s)\end{cases} \tag{3}$$

In order to find out the relation between the input force and angular variation, rearrange the first equation of the system (3), then work out the following equation.

$$X(s)=\left[\frac{(I+ml^2)}{ml}-\frac{g}{s^2}\right]\Phi(s) \tag{4}$$

Subsequently, substitute (4) for the term *X(s)* in the second equation of system (3), we can get:

$$\begin{aligned}(M+m)&\left[\frac{(I+ml^2)}{ml}-\frac{g}{s^2}\right]\Phi(s)s^2\\&+b\left[\frac{(I+ml^2)}{ml}-\frac{g}{s^2}\right]\Phi(s)s-ml\Phi(s)s^2=U(s)\end{aligned} \tag{5}$$

The relation between the input force and angular variation can be found out as (6).

$$\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s}{s^3+\frac{b(I+ml^2)}{q}s^2-\frac{(M+m)mgl}{q}s-\frac{bmgl}{q}} \tag{6}$$

Where,

$$q=[(M+m)(I+ml^2)-(ml)^2]$$

We will carry out a numeric simulation for dDIP, and an array of parameters of dDIP in practice should be substituted for the variables in the transfer functions accordingly.

These parameters of dDIP are listed as follows; they have the same definition with the simplified models. Assume that the acceleration of gravity g is $9.8m/s^2$.

M	0.43kg
m	0.1kg
l	0.25m
I	$0.0021kgm^2$

Substitute these parameters for the variables, and we can get the value of q.

$$\begin{aligned}q&=[(M+m)(I+ml^2)-(ml)^2]\\&=[(0.43+0.1)(0.0021+0.1\times0.25^2)-(0.1\times0.25)^2]\\&=0.0038\end{aligned}$$

Then substitute q and the other parameters for the corresponding variables in (6), the first transfer function.

$$\frac{\Phi(s)}{U(s)}=\frac{6.58}{s^2-34.17} \tag{7}$$

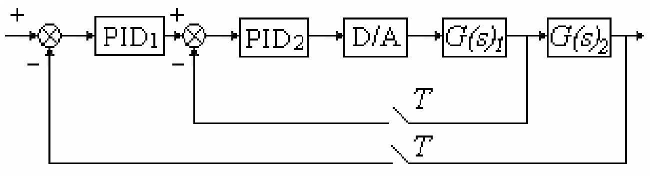

Fig.2 diagram of cascade PID control

On same principle, we can find the second function.

$$\frac{X(s)}{\Phi(s)} = \frac{0.402s^2 - 9.8}{s^2} \qquad (8)$$

Equation (7) and (8) are the transfer functions of dDIP, they will be used as models during the numeric simulating.

III. PID Controller and Simulation

It was mentioned that we had regarded the dDIP as a single input and double output system; therefore, it seemed that PID strategy cannot solve the problem properly as a kind of classical control theory, yet still, we had a widespread-used method to make it possible, the cascade PID controller.

A typical cascade PID controller is shown as Fig.2

The two transfer functions, G(s)1 and G(s)2 respectively represent the input and output relations of input force between pendulum angle and cart position. Each function is applied with a typical PID control loop. As the diagram shown, $G(s)_1$ is contained in the inner loop, for the pendulum angle, which varies more frequently, should be arranged in the inner-loop; and as the motor table position varies much slower, $G(s)_2$ should be placed in the outer-loop. The pendulum angle and cart position are acquired by using rotary encoder and linear encoder respectively, as the system is controlled by a numeric controller, the whole control routine runs with a sample time *T*.

We built the PID controller with the tool of Simulink, and the flow diagram is shown in Fig.3. One point should be mentioned that the Integrator of a PID controller contributes nothing to the whole system's stability, for it

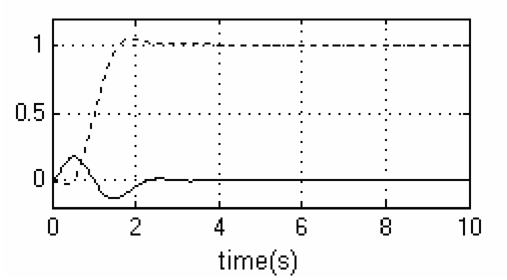

Fig. 4. Step response of the cart-pendulum system. The value of Y axis is only a numeric marker thus has no unit, and so as the Fig. 5.

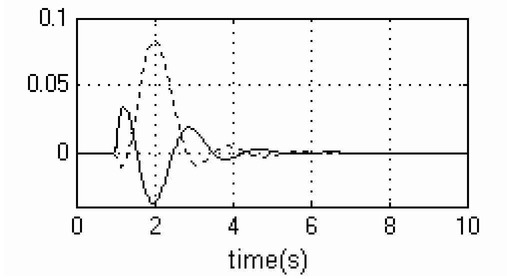

Fig. 5. Impulse response of the cart-pendulum system.

functionally produces unexpected tremendous overshoot when the linear motor runs in a relatively short traverse, therefore, the integrators in PID controller of the cascade system are abandoned.

In Fig.3, the model of dDIP is represented by the two functions near the end of the control flow. The final output value is given back and compared with the input value created at the last sample time, and then the result will be operated by PD link which serves as the outer loop controller; after that, the result will be compared with the feedback of the inner loop, which has been operated by another PD link; and finally, the result is passed to the two functions in a proper sequence. That produces the next output.

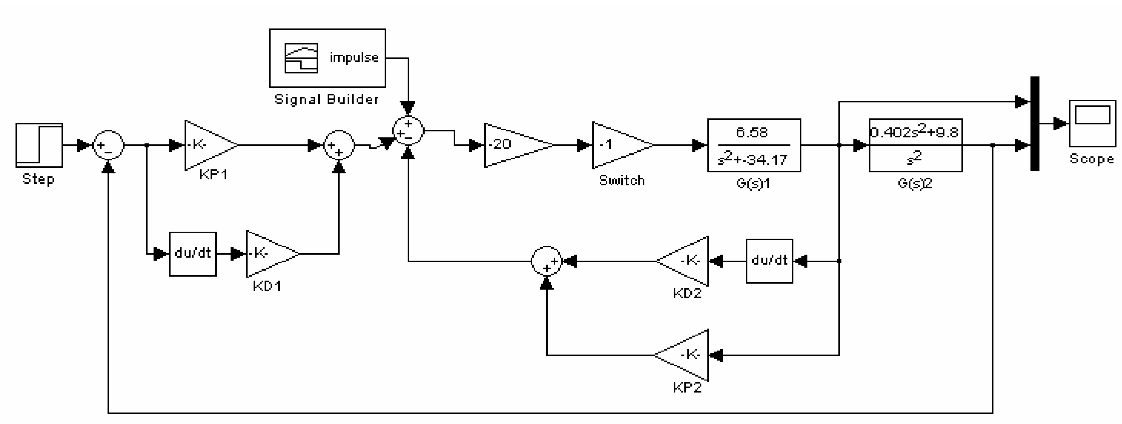

Fig. 3. Simulation block diagram of cascade PID control.

The simulation results are shown in the diagram Fig. 4 and Fig.5, according to the control parameters specified as KP1=0.24, KD1=0.24, KP2=1.65, and KD2=0.18.

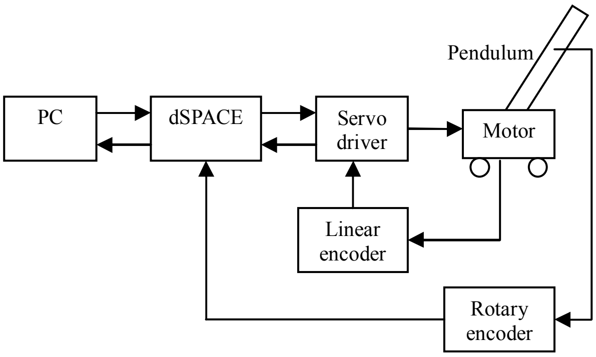

Fig. 6. Hardware linkage-diagram of the dDIP and its control system.

Fig.4 shows the process of the unit step response of cart. The dash curve shows the act of cart, and the solid curve shows the act of pendulum. This plot shows that after a step signal had applied to the system, the cart began to move and stopped at a new position, and meanwhile, the pendulum swayed several times and finally reached its upper equilibrium point.

Fig.5 shows the process of the impulse response of pendulum. The curves have the same definitions in Fig.4. The impulse is applied at the time of 1s and held on for 0.01s. The cart-pendulum system underwent a short period of oscillation and finally returned the original state.

We can draw a conclusion that the cascade PD controller is able to stabilize the cart-pendulum system. That is, when the pendulum is standing upright, the cart's position is controllable. In order to verify the simulation result, we organized an experiment in practice, using our dDIP, certainly.

IV. Experiment Results

We designed and made a linear motor before the experiment, and fixed the pendulum, pedestal and other accessories on the table of the motor. During the testing operation, the dSPACE controller is adopted. The hardware link diagram is shown in Fig.6.

The procedure of dSPACE control should be illustrated as follows. When the system gets started, a sample routine will acquire thevalue from the two encoders and send them to dSPACE, and then they will be attenuated respectively in order to keep in the same order of magnitude, for the original readings of the linear encoder is much bigger than the rotary encoder's; after that, the cascade PD operation will be applied, and finally the DAC block of dSPACE will send the signal in the form of voltage to the motor driver, which always amplifies the control signal it receives. In this way, the linear motor moves within a small range of distance, and keeps the pendulum standing upright.

The realistic control routine can be built simply by modifying the simulation part that previously stated, shown as Fig. 7.

The DS1104ENC_POS_C1 block is the linear encoder interface of dSPACE while the DS1104ENC_POS_C2 block is the rotary encoder interface; in addition, a DAC block should be provided because the dSPACE produces the output in the form of digital but the servo driver gets the signal in the form of voltage. Obviously, only the transfer functions are deleted and the above blocks are added to the controller. The trunk of PID controller used in simulation operation is not changed.

The last work is to setting the control parameters and see if the dDIP can reach its upper equilibrium point. We firstly increased the proportion factor KP1 of the outer loop, until the dDIP took on a tendency of oscillation; then increased the differential factor KD1 of the outer loop to make the system less unstable, repeat the same routine when tuning the inner loop PD controller. Generally, it was not difficult and time-consuming for us to let the dDIP work well.

Fig. 8 shows the dDIP working around its equilibrium point. Fig. 9 shows the variations of the motor table (cart) of the linear motor. Fig. 10 shows the angle of the pendulum.

Generally, the dDIP, which is controlled by PID strategy, takes on satisfying properties. By Fig. 9 and Fig. 10, one can

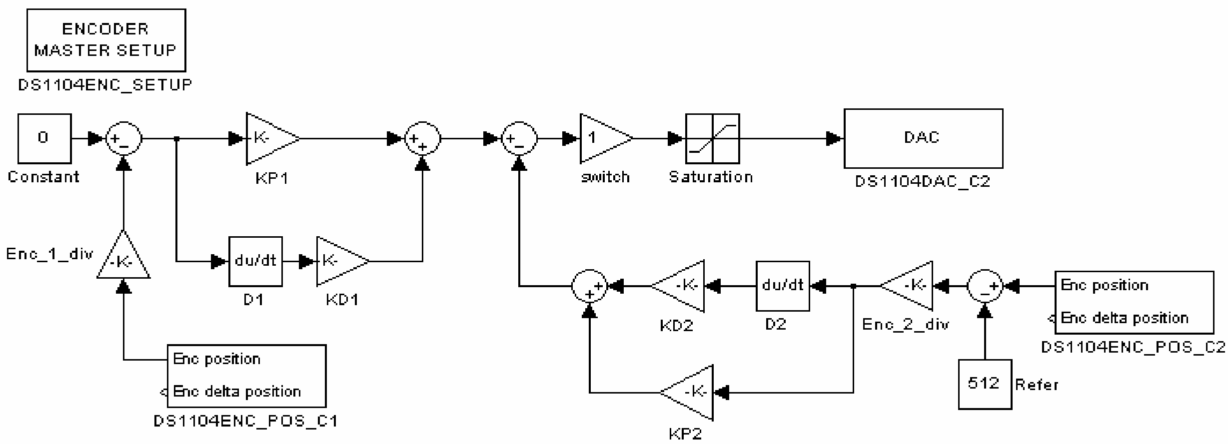

Fig. 7. dSPACE-based dDIP control flow diagram.

simply figure out that the dDIP's cart, the motor table, can easily hold its position within a small range of ±0.7mm and its pendulum angle within about $\pm 2.30 \times 10^{-3}$rad.

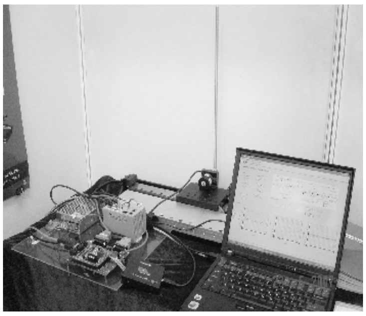

Fig. 8. The dDIP controlled by PID strategy

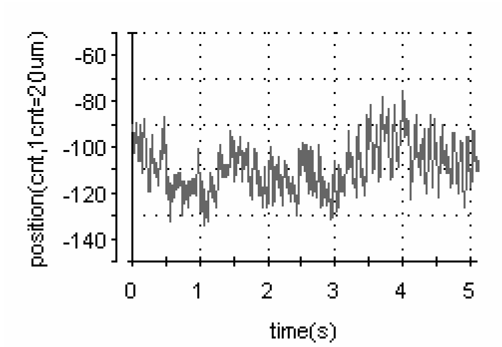

Fig. 9. the position variation of the motor table, which varied within a small range.

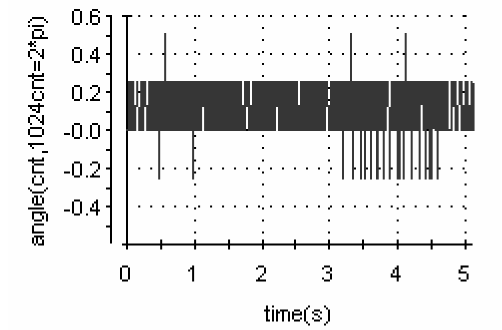

Fig. 10. the angle variation of the pendulum, it oscillated frequently around its upper equilibrium point.

V. Conclusion

The results of the experiment showed that PID strategy could effectively control the dDIP, and its parameters were easy to tune. Sometimes the dDIP could even reach a state of "totally static", which was shown as that the pendulum and the motor table kept still, thus the features of dDIP are much better than those conventional inverted pendulums which are applied with the similar control strategies. The further study may include the following aspects; to apply different strategies to dDIP in order to find out its further characteristics, and to transplant the multi-link pendulum to the platform of linear motor in order to discover its variation of controllability.

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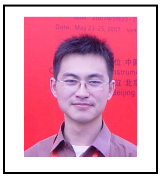

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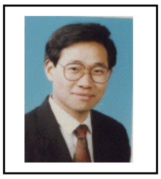

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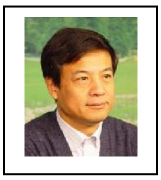

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