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BRIEF COMMUNICATION

Study of Energy Confinement Time by the Analytical Solution of Grad–Shafranov Equation with Lithium Limiter for Circular Cross-Section Tokamak

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Abstract In this work we calculated the energy confinement time by analytical solution of Grad–Shafranov equation (GSE) with Lithium limiter for circular cross-section HT-7 tokamak. A generalized Grad–Shafranov-type equation has been used. Specific functional forms of plasma internal energy and current are used. For this, the Shafranov parameter (asymmetry factor) and poloidal beta were obtained from by analytical solution of GSE. Than we can find the plasma energy confinement time. It is observed, the energy confinement time obtained from the analytical solution of GSE by using liquid lithium limiter is longer than that using graphite limiter, which shows that the plasma performance was improved.

Keywords Energy confinement time · Grad–Shafranov equation · Internal energy · Liquid lithium

Introduction

In tokamaks the plasma configurations are described in terms of solutions of the Grad–Shafranov equation (GSE). In the usual GSE [1, 2], the internal energy of plasma does not appear, but the internal energy [3] appears as a quantity to be determined. Analytical solutions of the Grad–Shafranov [1–3] equation are very useful for theoretical studies of plasma equilibrium, transport and magnetohydrodynamic (MHD) stability. In tokamaks, confinement of hot plasmas by means of strong magnetic fields is important research area. Confinement time, one of the main parameters of the ignition condition, is limited by thermal conduction and convection processes, but radiation is also a source of energy loss. The maximum energy confinement time can be determined by the microscopic behavior of the plasma such as collisions and microinstabilities [4]. This behavior ultimately leads to macroscopic energy transport, which can be either classical or anomalous depending on the processes involved. In the absence of instabilities, the confinement of toroidally symmetric tokamak plasma is determined by Coulomb collisions [4]. Since these phenomena require a knowledge of individual particles motion on short length scales and time scales, they are usually treated by kinetic models, but including only limited geometry because of the complexity of the physics. To achieve the thermonuclear condition (the ignition condition) in a tokamak ($nT\tau_E > 3 \times 10^{21}$ m^{-3} keVs), it is necessary to confine the plasma for a sufficient length of time. The global energy confinement time τ_E, is defined by $\int 3/2\, n(T_e + T_i)\frac{d^3x}{P}$, where n is the plasma density, T is the plasma particle temperature and P is the total input power [5–17].

Lots of lithium experiments have been carried out in tokamaks for the enhancement of plasma. In TFTR [18], plasmas with peaked density profile, high plasma current and long energy confinement time were achieved by injection of lithium pellets into plasma. In CDX-U [19–21], plasma discharges with lower loop voltage, wall recycling and edge oxygen and carbon radiation, and higher core electron temperature was achieved by using a toroidal liquid lithium limiter. In FTU and T-11 M, some excellent

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results were also obtained by application of liquid lithium limiters with capillary-pore system (CPS) structure [22–25].

To deal with lithium as PFMs and to provide databases for future research of lithium application as first wall, a movable lithium limiter [25] was designed on circular cross-section tokamak HT-7. By the first application of liquid lithium limiter, some positive results are obtained.

In this work we calculated the energy confinement time by analytical solution of Grad–Shafranov equation (GSE) with Lithium limiter for circular cross-section HT-7 tokamak [26, 27]. A generalized Grad–Shafranov-type equation has been used. Specific functional forms of plasma internal energy and current are used. For this, the Shafranov parameter (asymmetry factor) and poloidal beta were obtained from by analytical solution of GSE. Than we can find the plasma energy confinement time. It is observed, the energy confinement time obtained from the analytical solution of GSE by using liquid lithium limiter is longer than that using graphite limiter, which shows that the plasma performance was improved. Also, the energy confinement time was measured using the diamagnetic loop [25, 27]. Results of the two methods are in good agreement with each other.

Extended Grad–Shafranov Equation

Maxwell's equations together with the force balance equation from MHD equations, in the cylindrical coordinates (R, Z) reduce to the two-dimensional, nonlinear, elliptic partial differential equation, or GSE [13]: As in the linear case, the procedure to derive the GSE can be followed obtaining an extended GSE [3]

$$\Delta^{*}\psi = \mu_0 R J_{\varphi} = -\mu_0(\gamma-1)R^2\frac{du(\psi)}{d\psi} - F(\psi)\frac{dF(\psi)}{d\psi} \quad (1)$$

where Δ^{*} is

$$\Delta^{*} = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial z^2} \quad (2)$$

The internal energy $u(\psi)$ in this extended GSE is a function of ψ. The $u(\psi)$ and $F(\psi)$ are two free functions, and where μ_0 and J are the vacuum permeability and plasma current density respectively.

Energy Confinement Time by the Analytical Solution of Grad–Shafranov Equation

For circular cross-section HT-7 tokamak [26, 27], which is the ohmically heated tokamak, the GSE is solved by formally expanding as follows [13, 28]:

$$\psi(r,\theta) = \psi_0(r) + \psi_1(r)\cos\theta + \ldots, \quad (3)$$

$$u(\psi) = u_2(\psi_0) + \frac{du_2(\psi_0)}{d\psi_0}\psi_1\cos\theta + \ldots, \quad (4)$$

$$F(\psi) = RB_{\varphi} = R_0[B_0 + B_{\varphi 2}(\psi_0) + \ldots], \quad (5)$$

where $B_0 = const$, is the vacuum toroidal field at $R = R_0$, $B_{\varphi 2}(\psi)$ is a new free function replacing $F(\psi)$. In the first order solution or toroidal force balance approximation and if plasma were surrounded by a perfectly conducting shell located at $r = b$, then first order flux function [13, 28] is:

$$\psi_1(r) = B_{\theta 1}(r)\int_r^b \frac{dx}{xB_{\theta 1}^2(x)} \times \int_0^x \left[2\mu_0(\gamma-1)y^2\frac{du_2(y)}{dy} - yB_{\theta 1}^2(y)\right]dy \quad (6)$$

where $B_\theta = \frac{\mu_0 I_p}{2\pi r}$

If there are external coils to produce vertical magnetic field, the boundary condition on the flux function is modified so that we have [13]:

$$\psi(b,\theta) = const. + \psi_v(b,\theta), \quad (7)$$

where $\psi_v(r, \theta) = R_0 B_v r\cos\theta$, is the flux function due to external vertical field coils and therefore the full toroidal correction to ψ is [13, 28]:

$$\psi_1(total) = \psi_{1T}(r)\cos\theta = \left[\psi_1(r) + \left[\frac{bR_0B_v}{B_{\theta 1}(b)}\right]B_{\theta 1}(r)\right]\cos\theta \quad (8)$$

The shift of the plasma column center from the geometrical center of vacuum chamber given by [13, 28]:

$$\Delta R = -\frac{\psi_{1T}(a)}{\psi_0'(a)} = -\frac{\psi_1(a)}{\psi_0'(a)} - \Delta R_v = -\frac{\psi_1(a)}{\psi_0'(a)} - \frac{bB_v}{B_{\theta 1}(b)} \quad (9)$$

where $B_{\theta 1}(b) = \frac{\mu_0 I_p}{2\pi b}$.

Therefore, the first relation for plasma position [13, 28] is

$$\Delta R_{Analytical} = \frac{b^2}{2R_0}\times\left[\left(\beta_p + \frac{l_i - 1}{2}\right)\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a}\right] - \frac{bB_v}{B_{\theta 1}(b)} \quad (10)$$

where β_p is the poloidal beta, l_i is the internal inductance of the plasma, and B_v is the average vertical magnetic field over the vacuum chamber. We can find B_v from saddle sine coil [27] and expression $\beta_p + \frac{l_i}{2}$ from magnetic coils measurement [27, 29]:

$$\beta_p + \frac{l_i}{2} = 1 + \ln\frac{a}{b} + \frac{\pi R_0}{\mu_0 I_0}(\langle B_\theta\rangle + \langle B_n\rangle), \quad (11)$$

where

$$\langle B_\theta\rangle = B_\theta(\theta = 0) - B_\theta(\theta = \pi),$$

$$\langle B_n\rangle = B_n(\theta = \frac{\pi}{2}) - B_n(\theta = \frac{3\pi}{2}), \quad (12)$$

We measured these local magnetic fields with magnetic probes [27] at above angles.

Before we derive the relation for the energy confinement time, we must determine the volume-averaged plasma kinetic pressure $\langle p\rangle$, and then the plasma thermal energy U. $\langle p\rangle$ can be determined directly from the definition of the poloidal beta [4]:

$$\langle p\rangle = \beta_p \frac{B_\theta^2(a)}{2\mu_0} = \mu_0 \frac{I_p^2 \beta_p}{8\pi^2 a^2} \quad (13)$$

where a is the plasma minor radius. For the measurement of the plasma thermal energy, we start from the plasma state equation [4]:

$$\langle p\rangle = \sum\nolimits_i n_i T_i = \frac{2}{3}\sum\nolimits_i E_i = \frac{2}{3}E, \quad (14)$$

where subscript 'i' indicates the plasma species i and E indicates the plasma thermal energy density; therefore the plasma thermal energy U and the plasma temperature are obtained [4]:

$$U = \frac{3}{2}\left(\sum\nolimits_\alpha n_\alpha T_\alpha\right)V = \frac{3}{2}\langle p\rangle V, \quad (15)$$

where V is the plasma volume. The plasma-specific resistance in the steady state plasma can be written as [4]

$$\rho_p = \frac{1}{\sigma_p} = \frac{A}{l}R_p = \frac{a^2}{2R_0}\frac{V_R}{I_p} \quad (16)$$

where σ_p is the plasma conductivity, R_p is the plasma resistance and V_R is the resistive component of the loop voltage (the poloidal flux loop).

But the most important of these measurements is determining the plasma thermal energy confinement time, which is defined by [4]

$$\frac{dU}{dt} = P_{Ohmic} - \frac{U}{\tau_E}, \quad (17)$$

where τ_E is the plasma energy confinement time and P_{Ohmic} is the rate of input heating power. Rearranging Eq. (17), the ohmic heating power is [4]

$$P_{Ohmic} = V_R I_p - \frac{d}{dt}\left(\frac{1}{2}LI_p^2\right), \quad (18)$$

If the plasma is in thermal equilibrium ($L' = 0$ and $I' = 0$), then from Eqs. (15) and (17), we have

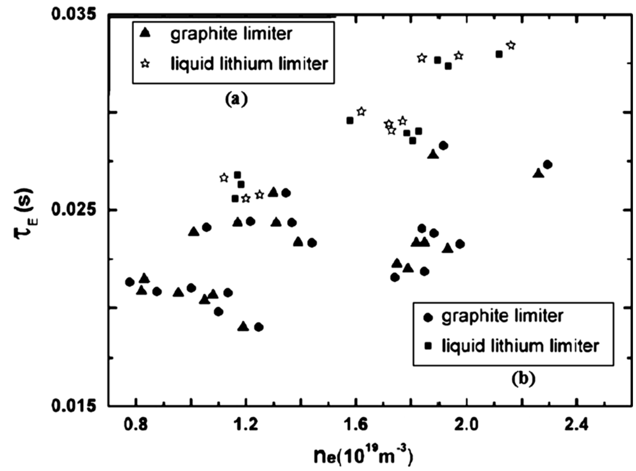

Fig. 1 Energy confinement time, obtained (*a*) Experimental [25] and (*b*) analytical by solution of GSE as a function of line averaged electron density

$$P_{Ohmic} = V_R I_p = R_p I_p^2 = \frac{U}{\tau_E}, \quad (19)$$

$$\tau_E = \frac{3}{8}\mu_0 R_0 \frac{I_p \beta_p}{V_R} = \frac{3}{8}\mu_0 R_0 \frac{\beta_p}{R_p}, \quad (20)$$

Also if $\frac{dU}{dt}$ is not negligible, then from Eqs. (15) and (17), we have

$$\frac{1}{\tau_E} = \frac{8}{3}\frac{R_p}{\mu_0 R_0 \beta_p} - \frac{2I'}{I} - \frac{\beta_p'}{\beta_p}, \quad (21)$$

Therefore, according to the above discussion, the Shafranov parameter (asymmetry factor) and poloidal beta were obtained from by analytical solution of GSE. Than we can find the plasma energy confinement time.

It is observed from Fig. 1, the energy confinement time obtained from the analytical solution of GSE by using liquid lithium limiter is longer than that using graphite limiter, which shows that the plasma performance was improved. Also, the energy confinement time was measured using the diamagnetic loop [25, 27]. Results of the two methods are in good agreement with each other.

Conclusion

We present the calculation of the energy confinement time by analytical solution of Grad–Shafranov equation (GSE) with Lithium limiter for circular cross-section HT-7 tokamak. A generalized Grad–Shafranov-type equation has been used. Specific functional forms of plasma internal energy and current are used. For this, the Shafranov parameter (asymmetry factor) and poloidal beta were obtained from by analytical solution of GSE. Than we can find the plasma energy confinement time. It is observed, the energy confinement time obtained from the analytical solution of GSE by using liquid lithium limiter is longer than that using graphite limiter, which shows that the plasma performance was improved. Also, the energy confinement time was measured using the diamagnetic loop [25, 27]. Results of the two methods are in good agreement with each other.

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