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Beam wander of laser beam propagating through oceanic turbulence

Lu Lu[a,b], Zhiqiang Wang[a,b], Pengfei Zhang[a], Jinghui Zhang[a], Xiaoling Ji[c], Chengyu Fan[a,*], Chunhong Qiao[a]

[a] Key Laboratory of Atmospheric Composition and Optical Radiation, Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei, 230031, China
[b] University of Science and Technology of China, Hefei, 230026, China
[c] Department of Physics, Sichuan Normal University, Chengdu, 610066, China

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ABSTRACT

The analytical expressions for beam wander of collimated and focused beam in oceanic turbulence are derived. Compared with the previously integrating ones, it can be seen that the two results are in agreement with each other exactly for the collimated and focused beam, respectively. Further, the influences of three main oceanic parameters (i.e., the rate of dissipation of mean-squared temperature χ_T, the rate of dissipation of kinetic energy per unit mass of seawater ε and the ratio of temperature to salinity contribution to the refractive index spectrum w) and the beam radius W_0 on beam wander are investigated in the collimated and focused beam cases. The results indicate that the beam wander increases as ε decreases, χ_T increases, salinity-induced predominates and W_0 decreases in mentioned above cases. In addition, based on the dimensionless quantity B_W, the relation between beam wander and the long-term spot size or turbulence-induced beam spot size is investigated. In particular, to distinguish beam wander among different beam types, the relative beam wander is defined. Based on this definition, the increment of beam wander between focused and collimated beam is larger than that of arbitrary beam type (i.e., $0 < \Theta_0 < 1$). It is beneficial to select the predominant beam type in laser propagation applications

1. Introduction

Movement of the short-term beam instantaneous center (or "hot spot") is commonly called beam wander [1]. This phenomenon can be characterized statistically by the variance of the hot spot displacement along an axis or by the variance of the magnitude of the hot spot displacement [1]. An estimate of the short-term beam radius is obtained by removing beam wander effects from the long-term beam radius [1]. It is much convenient to use the geometrical optics approximation method in the turbulent area. Beam wander is an important characteristic of laser beams, which determines their utility for practical applications, such as laser communication [2,3], global quantum communication [4]. Until now, beam wander analysis in an optical ground station-satellite uplink has been reported [2]. Influence of beam wander on uplink of ground-to-satellite laser communication and optimization for transmitter beam radius has been researched [3]. Berman et al. have discussed the influence of the initial spatially coherent length on the beam wander [5]. The beam wander of various beam has been investigated, such as dark hollow beam, flat-topped beam, annular beam, cosh-Gaussian beam, Bessel beam, twin thin beam, electromagnetic Gaussian-Schell beam, Gaussian Schell-model beam, quantization Gaussian beam and Airy beam [6–13]. Compared to the optical propagation through the turbulent atmosphere, light propagation through seawater is relatively unexplored. Since the power spectrum of oceanic turbulence proposed by Nikishov and Nikishov [14], there has been remarkable interest in the study of propagation characteristics using laser beams in seawater. The power spectrum of oceanic turbulence has been simplified for homogeneous and isotropic water media in Ref. [15]. This spectrum is applicable for isothermal water and invalid for dominating salinity-induced optical turbulence (i.e., $w = 0$) in detail [16]. Since recently the interest in active optical underwater communications, imaging and sensing appeared [17,18], it has become important to deeply understand how the oceanic turbulence affects laser propagation [19–22]. Recently, the effect of polarization characteristics [20], the intensity and coherence properties of light [21] and light scintillation [22] in oceanic turbulence have been studied, respectively. Very recently, the wave structure function and the radial Gaussian laser array beams propagating in oceanic turbulence have also been researched [16,23]. However, to the best of our knowledge, beam wander of laser beam propagating through oceanic turbulence has not been reported. In

* Corresponding author.
E-mail address: cyfan@aiofm.ac.cn (C. Fan).

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this paper, based on the oceanic power spectrum, we analyze the beam wander effect with analytical and numerical methods. It is believed that these results can be used in practical applications.

2. Beam wander of laser beam

The far-field angular spread of a free-space propagating beam of diameter $2W_0$ is of order $\lambda/2W_0$. In the presence of optical turbulence, a finite optical beam will experience random deflections as it propagates, causing further spreading of the beam by large-scale inhomogeneities of the turbulence [1]. Over short time periods the beam profile at the receiver moves off the boresight and can become highly skewed from Gaussian so that the instantaneous center of the beam is randomly displaced [1]. According to Ref. [1], W^2T_{LS} describes the beam wander or the variance of the instantaneous center of the beam in the receiver plane ($z = L$).

Based on the introduction of a general model [1], beam wander can be expressed as

$$a\left\langle r_c^2\right\rangle = W^2T_{LS} = 4\pi^2k^2W^2\int_0^L\int_0^\infty \kappa\Phi_n(\kappa)H_{LS}(\kappa,z)[1-\exp(-\Lambda L\kappa^2\xi^2/k)]\mathrm{d}\kappa\,\mathrm{d}z, \tag{1}$$

where <> denotes an ensemble average, k is the wave number related to the wavelength λ by $k=2\pi/\lambda$. W, Λ represents the beam radius in the free space and Fresnel ration of beam at receiver, respectively. $\Phi_n(\kappa)$ is the power spectrum of turbulence, $H_{LS}(\kappa, z)$ is the large-scale filter function, κ is the magnitude of spatial wave number, the normalized distance variable $\xi = 1 - z/L$.

According to Ref. [20], when the eddy thermal diffusivity and the diffusion of salt are assumed to be equal, the power spectrum for homogeneous and isotropic oceanic water is given by the expression

$$\Phi_n(\kappa) = 0.388\times10^{-8}\varepsilon^{-1/3}\chi_T\kappa^{-11/3}[1+2.35(\kappa\eta)^{2/3}] \times[w^2\exp(-A_T\delta)+\exp(-A_S\delta)-2w\exp(-A_{TS}\delta)]/w^2, \tag{2}$$

where ε is the rate of dissipation of kinetic energy per unit mass of fluid ranging from $10^{-1}\,\mathrm{m^2/s^3}$ to $10^{-10}\,\mathrm{m^2/s^3}$ [20], and χ_T is the rate of dissipation of mean-squared temperature ranging from $10^{-4}\,\mathrm{K^2/s}$ to $10^{-10}\,\mathrm{K^2/s}$ [20]. w (unitless) is the ratio of temperature to salinity contribution to the refractive index spectrum, which in oceanic waters varies in the interval [−5; 0], with −5 and 0 corresponding to dominating temperature-induced and salinity-induced optical turbulence, respectively [20]. $\eta = 10^{-3}$m is the Kolmogorov micro scale (inner scale). Besides, $A_T = 1.863\times10^{-2}$, $A_S = 1.9\times10^{-4}$, $A_{TS} = 9.41\times10^{-3}$ and $\delta = 8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2$.

The large-scale filter function is [1]

$$H_{LS}(\kappa,\xi) = \exp\{-\kappa^2W_0^2[(\Theta_0+\bar{\Theta}_0\xi)^2+\Lambda_0^2(1-\xi)^2]\}, \tag{3}$$

where $\Theta_0 = 1-\bar{\Theta}_0$. W_0, Θ_0 and Λ_0 are the beam radius, the beam curvature parameter and Fresnel ration of beam at transmitter, respectively [1].

Because beam wander is caused mostly by a large-scale turbulence near the transmitter, the last term can be dropped in (3) and the geometrical optics approximation is [1]

$$1-\exp(-\Lambda L\kappa^2\xi^2/k) = \Lambda L\kappa^2\xi^2/k,\quad L\kappa^2/k \ll 1. \tag{4}$$

Substituting from Eqs. (2)–(4) into Eq. (1), Eq. (1) leads to

$$\left\langle r_c^2\right\rangle = 0.388\times10^{-8}\times4\pi^2kW^2L\Lambda\varepsilon^{-1/3}(\chi_T/w^2) \times\int_0^1\int_0^\infty\kappa^{-2/3}[1+2.35(\kappa\eta)^{2/3}] \times[w^2\exp(-A_T\delta)+\exp(-A_S\delta)-2w\exp(-A_{TS}\delta)] \times\exp[-\kappa^2W_0^2(\Theta_0+\bar{\Theta}_0\xi)^2]\xi^2\,\mathrm{d}\kappa\,d\xi, \tag{5}$$

then

$$\left\langle r_c^2\right\rangle = 0.388\times10^{-8}\times4\pi^2kW^2L\Lambda\varepsilon^{-1/3}(\chi_T/w^2)\int_0^1\xi^2\,d\xi \times\int_0^\infty(\kappa^{-2/3}+g)[w^2\exp(-a\kappa^{4/3}-b\kappa^2) +\exp(-c\kappa^{4/3}-d\kappa^2)-2w\exp(-e\kappa^{4/3}-f\kappa^2)]\,\mathrm{d}\kappa, \tag{6}$$

where $a = 8.284A_T\eta^{4/3}$, $b = 12.978A_T\eta^2+W_0^2\left(\Theta_0+\bar{\Theta}_0\xi\right)^2$, $c = 8.284A_S\eta^{4/3}$, $d = 12.978A_S\eta^2+W_0^2\left(\Theta_0+\bar{\Theta}_0\xi\right)^2$, $e = 8.284A_{TS}\eta^{4/3}$, $f = 12.978A_{TS}\eta^2+W_0^2\left(\Theta_0+\bar{\Theta}_0\xi\right)^2$, $g = 2.35\eta^{2/3}$.

After very tedious calculations, the beam wander is expressed as

$$\begin{aligned}\left\langle r_c^2\right\rangle &= 0.388\times10^{-8}\times4\pi^2kW^2L\Lambda\varepsilon^{-1/3}(\chi_T/w^2)\\
&\times\int_0^1\xi^2\left\{w^2b^{-1/6}\Gamma(1/6)(1-7a^3/216b^2)/2\right.\\
&+d^{-1/6}\Gamma(1/6)(1-7c^3/216d^2)/2\\
&-wf^{-1/6}\Gamma(1/6)(1-7e^3/216f^2)\\
&-w^2ab^{-5/6}\Gamma(5/6)(1-55a^3/864b^2)/2\\
&-cd^{-5/6}\Gamma(5/6)(1-55c^3/864d^2)/2\\
&+wef^{-5/6}\Gamma(5/6)(1-55e^3/864f^2)\\
&+w^2a^2b^{-3/2}\Gamma(3/2)(1-a^3/16b^2)/4\\
&+c^2d^{-3/2}\Gamma(3/2)(1-c^3/16d^2)/4\\
&-we^2f^{-3/2}\Gamma(3/2)(1-e^3/16f^2)/2\\
&+w^2gb^{-1/2}\Gamma(1/2)(1-a^3/8b^2)/2\\
&+gd^{-1/2}\Gamma(1/2)(1-c^3/8d^2)/2\\
&-wgf^{-1/2}\Gamma(1/2)(1-e^3/8f^2)\\
&-w^2gab^{-7/6}\Gamma(7/6)(1-91a^3/864b^2)/2\\
&-gcd^{-7/6}\Gamma(7/6)(1-91c^3/864d^2)/2\\
&+wgef^{-7/6}\Gamma(7/6)(1-91e^3/864f^2)\\
&+w^2ga^2b^{-11/6}\Gamma(11/6)(1-187a^3/2160b^2)/4\\
&+gc^2d^{-11/6}\Gamma(11/6)(1-187c^3/2160d^2)/4\\
&\left.-wge^2f^{-11/6}\Gamma(11/6)(1-187a^3/2160b^2)/2\right\}d\xi.\end{aligned} \tag{7}$$

where $\Gamma(\bullet)$ is the Gamma function and Eq. (7) is applicable for collimated, divergent or focused Gaussian beam. In our paper, we analyze two special cases (i.e., collimated beam and focused beam).

For collimated beam $\left(\Theta_0 = 1\right)$, Eq. (7) can be simplified as

$$\left\langle r_c^2\right\rangle_{coll} = 0.517\times10^{-8}\times\pi^2kW^2L\Lambda\varepsilon^{-1/3}(\chi_T/w^2) \times(\alpha w^2-2\beta w+\gamma), \tag{8}$$

Table 1
Parameters relating to Eqs. (8) and (10).

	$W_0 = 0.05$ m	$W_0 = 0.1$ m	$W_0 = 0.15$ m
α	7.96957	6.20393	5.37672
β	15.9407	12.4083	10.7537
γ	7.97116	6.20483	5.37694
α'	9.11932	7.05683	6.10055
β'	18.2430	14.1149	12.2017
γ'	9.12373	7.05803	6.10112

where

$$\begin{aligned}\alpha = {} & 37.9244\eta^4 A_T^2\left(W_0^2+12.978A_T\eta^2\right)^{-11/6} \\ & +15.2043\eta^{8/3}A_T^2\left(W_0^2+12.978A_T\eta^2\right)^{-3/2} \\ & -9.03014\eta^{8/3}A_T\left(W_0^2+12.978A_T\eta^2\right)^{-7/6} \\ & -4.67544\eta^{4/3}A_T\left(W_0^2+12.978A_T\eta^2\right)^{-5/6} \\ & +2.08263\eta^{4/3}\left(W_0^2+12.978A_T\eta^2\right)^{-1/2} \\ & +2.78316\left(W_0^2+12.978A_T\eta^2\right)^{-1/6}. \end{aligned} \tag{9}$$

and β, γ can be obtained from Eq. (9) if A_T is replaced by A_{TS}, A_S, respectively.

For focused beam $\left(\Theta_0=0\right)$, Eq. (7) can be simplified as

$$\begin{aligned}\left\langle r_c^2\right\rangle_{focu} = {} & 0.517\times10^{-8}\times\pi^2 kW^2L\Lambda\varepsilon^{-1/3}(\chi_T/w^2) \\ & \times(\alpha' w^2-2\beta' w+\gamma'), \end{aligned} \tag{10}$$

where

$$\begin{aligned}\alpha' = \int_0^1 {} & 37.9244\eta^4 A_T^2\left(W_0^2\xi^2+12.978A_T\eta^2\right)^{-11/6} \\ & +15.2043\eta^{8/3}A_T^2\left(W_0^2\xi^2+12.978A_T\eta^2\right)^{-3/2} \\ & -9.03014\eta^{8/3}A_T\left(W_0^2\xi^2+12.978A_T\eta^2\right)^{-7/6} \\ & -4.67544\eta^{4/3}A_T\left(W_0^2\xi^2+12.978A_T\eta^2\right)^{-5/6} \\ & +2.08263\eta^{4/3}\left(W_0^2\xi^2+12.978A_T\eta^2\right)^{-1/2} \\ & +2.78316\left(W_0^2\xi^2+12.978A_T\eta^2\right)^{-1/6}\xi^2\, d\xi. \end{aligned} \tag{11}$$

and β', γ' can be obtained from Eq. (11) when A_T is replaced by A_{TS}, A_S, respectively.

In this paper, substituting $A_T = 1.863\times10^{-2}$, $A_S = 1.9\times10^{-4}$, $A_{TS} = 9.41\times10^{-3}$ and $\eta = 10^{-3}$m into Eqs. (9) and (11), we choose the $W_0 = 0.05$ m, 0.1 m and 0.15 m, respectively. α, β, γ, α', β' and γ' can be simplified in Table 1.

To obtain the increment of beam wander among different beam types, we define the relative beam wander $\left\langle r_c^2\right\rangle_R$ based on focused and collimated beam, which can be expressed as

$$\begin{aligned}\left\langle r_c^2\right\rangle_R & = \left\langle r_c^2\right\rangle_{focu} - \left\langle r_c^2\right\rangle_{coll} \\ & = 0.517\times10^{-8}\times\pi^2 kW^2L\Lambda\varepsilon^{-1/3}(\chi_T/w^2)[(\alpha'-\alpha)w^2-2(\beta'-\beta)w+(\gamma'-\gamma)]. \end{aligned} \tag{12}$$

it is clear that the increment $\left\langle r_c^2\right\rangle_R' = \left\langle r_c^2\right\rangle_{focu} - \left\langle r_c^2\right\rangle_{arbi}$ or $\left\langle r_c^2\right\rangle_R' = \left\langle r_c^2\right\rangle_{arbi} - \left\langle r_c^2\right\rangle_{coll}$ of arbitrary beam type (i.e., $0 < \Theta_0 < 1$) is smaller than that of Eq. (12).

3. Numerical calculation results and analysis

In this part, $\eta = 10^{-3}$m, $\lambda = 0.417\mu$m, $W = 0.5$m, $\Lambda = 0.5$m and $L = 50$m are chosen. In order to examine the correctness of the analytical results obtained in this paper, we give a comparison of results of beam wander calculated by the analytical expressions in collimated and focused beam and by the integral of Eq. (5) in Figs. 1–2, respectively. It can be seen that the two results are in agreement with each other exactly with the fixed oceanic parameters. Fig. 1(a) and (b), beam wander for collimated beam increases as $\log\varepsilon$ decreases, and $\log\chi_T$ increases. Beam wander is affected significantly when the salinity-induced dominates in Fig. 1(c). In focused beam case, the results are similar to collimated beam except that beam wander for focused beam is larger than collimated one in Fig. 2(a)–(c). Besides, the influence of beam wander effect is determined by the value of beam radius at transmitter W_0 and the smaller beam radius leads to the larger beam wander in two cases (i.e., collimated and focused beam).

Furthermore, the relation between beam wander and the turbulence-induced beam spot size is investigated in detail by using numerical and theoretical methods. Using the dimensionless quantity $B_W = \left\langle r_c^2\right\rangle/W^2(1+T)$, where $T = 4\pi^2k^2L\int_0^1\int_0^\infty \kappa\Phi_n(\kappa)(1-e^{-\Lambda L\kappa^2\xi^2/k})\,d\kappa\, d\xi$ [1]. Based on Ref. [23], in oceanic turbulence, the quantity can be expressed as $T = 0.517\times10^{-8}\pi^2kL^2\Lambda\varepsilon^{-1/3}\chi_T(67.832w^2-176.699w+475.692)/w^2$. For the dimensionless quantity, B_W is more informative than merely $\left\langle r_c^2\right\rangle$ about the practical significance of the beam wander. The quantity B_W can be used to investigate the proportion of beam wander $\left\langle r_c^2\right\rangle$ to the turbulence-induced beam spot size $W^2(1+T)$ [1]. In Fig. 3, the dimensionless quantity B_W as a function of the three oceanic parameters is plotted. It is shown that the larger value of B_W is related to smaller $\log\varepsilon$, larger $\log\chi_T$ and salinity-induced dominant. The beam wander of collimated beam has less influence on turbulence-induced beam spot size compared to that of focused beam. Furthermore, beam wander plays an important role in turbulence-induced beam spot size because all the values of B_W are larger than 0.034 (or 3.4%) in Fig. 3. The beam wander effect cannot be ignored on laser beam propagating through ocean.

As we know, it is impossible to avoid the beam wander effect for laser propagation. Therefore, achieving small beam wander is imperative. In this paper, the relative beam wander describes the increment of beam wander between focused and collimated beam, and this quantity benefits us to obtain small value of beam wander. Based on Eq. (12), it is feasible to obtain small beam wander as long as the arbitrary beam type is under $\Theta_0\to1$ condition. Choosing the same turbulent strength and beam radius (i.e., $\varepsilon = 10^{-5}$m^2/s^3, $\chi_T = 10^{-7}$K^2/s, $w = -3$ and $W_0 = 0.05$m) in Figs. 2 and 4, the value of $\left\langle r_c^2\right\rangle_R/\left\langle r_c^2\right\rangle_{focu}$ is equal to 0.124. For arbitrary beam, it is clear that the ratio of $\left\langle r_c^2\right\rangle_R'/\left\langle r_c^2\right\rangle_{focu}$ is smaller than 0.124. The extension to arbitrary oceanic parameters and beam radius is straightforward. In Fig. 4, it is also shown that the larger beam radius leads to smaller value of beam wander. In the practical applications, it is a feasible method to achieve small beam wander effect when beam curvature parameter at transmitter Θ_0 is approximate to 1 and beam radius is large. It is also meaningful to select proper beam parameters which are less sensitive to turbulence in laser propagation applications.

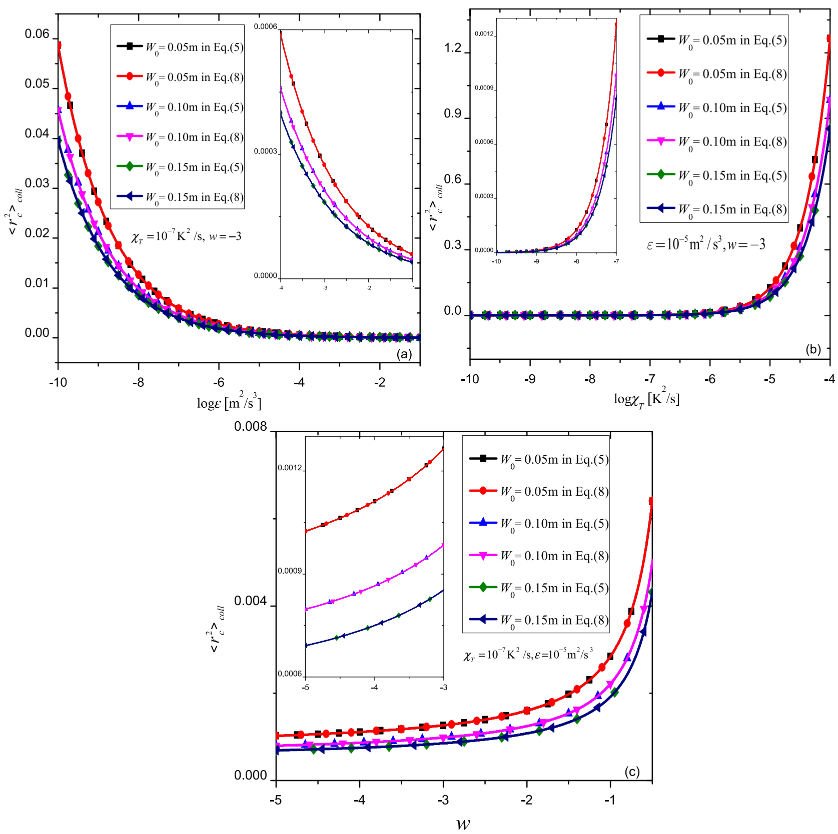

Fig. 1. Changings of beam wander for collimated beam versus (a) log ε, (b) log χ_T and (c) w.

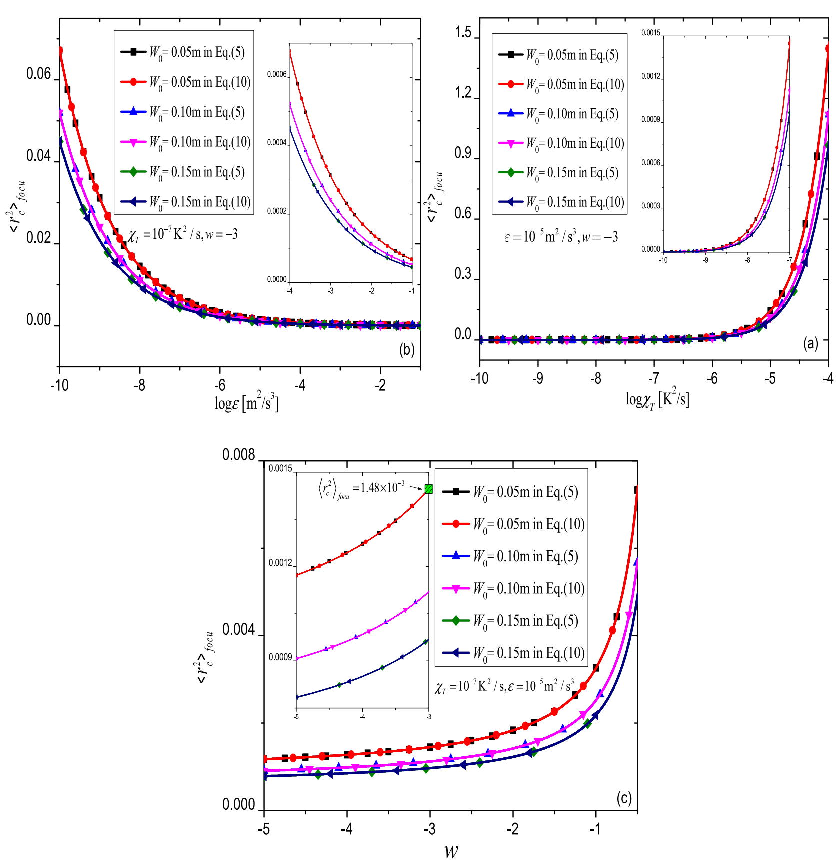

Fig. 2. Changings of beam wander for focused beam versus (a) log ε, (b) log χ_T and (c) w.

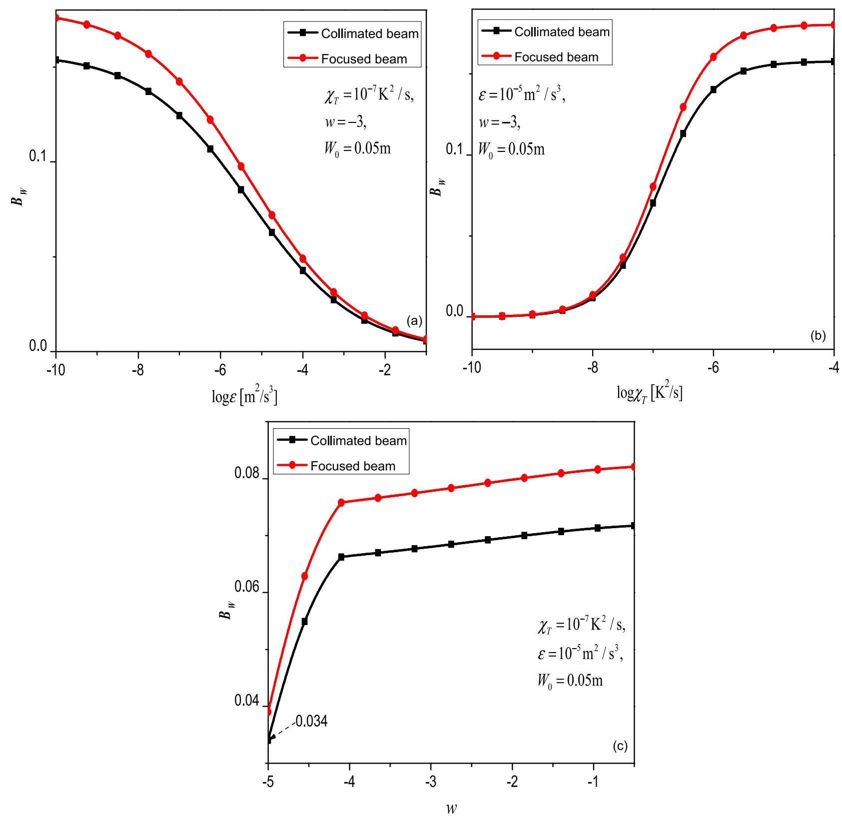

Fig. 3. Dimensionless quantity B_W for collimated and focused beam versus (a) $\log\varepsilon$, (b) $\log\chi_T$ and (c) w.

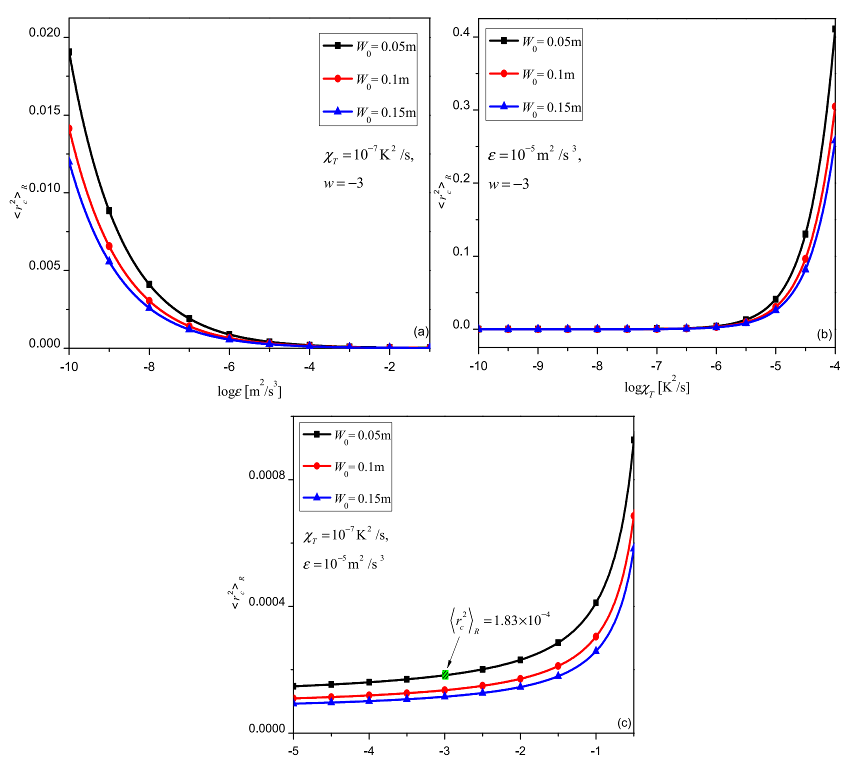

Fig. 4. Relative beam wander with various beam radius versus (a) $\log\varepsilon$, (b) $\log\chi_T$ and (c) w.

4. Concluding remarks

In this paper, based on the oceanic power spectrum, we study the beam wander effect with analytical and numerical methods in weak fluctuation theory, and derive the analytical expressions for beam wander of collimated and focused beam in oceanic turbulence. Giving a comparison of results of the analytical expression and the previously obtained integral form, it can be seen that the results of analytical expression are in agreement with each other exactly. Further, the influences of three oceanic parameters (i.e., ε, χ_T and w) and beam radius W_0 on beam wander are investigated for collimated and focused beam. The results indicate that the beam wander increases as ε decreases, χ_T increases, salinity-induced dominates and W_0 decreases both in collimated-beam and focused-beam cases. The difference of beam wander between focused and collimated beam is that a collimated beam has smaller value. Besides, the smaller beam radius at transmitter leads to the stronger beam wander effect. In addition, based on the dimensionless quantity B_W, the relation between beam wander and the turbulence-induced beam spot size is investigated. It is shown that the beam wander of collimated beam has the less influence on turbulence-induced beam spot size compared to that of focused beam. Particularly, according to the definition of the relative beam wander, it is shown that the relative beam wander is small when the value of beam curvature parameter at transmitter Θ_0 is close to 1 (i.e., $\Theta_0 \to 1$) and beam radius W_0 is properly large. It is believed that these results may be used in practical applications.

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