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A generalization of the Drude-Smith formula for magneto-optical conductivities in Faraday geometry

F. W. Han,[1,2] W. Xu,[1,2,3,a)] L. L. Li,[1] and C. Zhang[1]
[1]*Key Laboratory of Materials Physics, Institute of Solid State Physics, Chinese Academy of Sciences, Hefei 230031, China*
[2]*University of Science and Technology of China, Hefei 230026, China*
[3]*Department of Physics and Astronomy and Yunnan Key Laboratory for Micro/Nano Materials and Technology, Kunming 650091, China*

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In this study, we generalize the impulse response approach and Poisson statistics proposed by Smith [Phys. Rev. B **64**, 155106 (2001)] to evaluate the longitudinal and transverse magneto-optical conductivities in an electron gas system in Faraday geometry. Comparing with the standard Drude model, the coefficients a_n are introduced in the Drude-Smith formula to describe the backscattering or localization effect for the nth electronic scattering event. Such a formula can also be applied to study the elements of the dielectric function matrix in the presence of magnetic and radiation fields in electron gas systems. This theoretical work is primely motivated by recent experimental activities in measuring the real and imaginary parts of longitudinal and transverse magneto-optical conductivities in condensed matter materials and electronic devices using terahertz time-domain spectroscopy. We believe that the results obtained from this study can provide an appropriate theoretical tool in reproducing the experimental findings and in fitting with experimental data to determine the important sample and material parameters. *Published by AIP Publishing.*
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I. INTRODUCTION

For good metals and semiconductors, the frequency dependence of the complex optical conductivity in the absence of an external magnetic field can be described by the well-known Drude formula: $\sigma(\omega) = \sigma_0/(1 - i\omega\tau)$, where ω is the radiation frequency, τ is the electronic relaxation time, and the DC conductivity σ_0 is given as $n_e e^2\tau/m^*$ with n_e being the electron density and m^* the effective mass of an electron in the material. The basic feature of the Drude formula is that it suggests a maximum of the real part of the conductivity at zero frequency and the real part of the conductivity decreasing with increasing ω in a Lorentzian form. However, in some electrically conductive condensed matter systems, especially in poor metals[1,2] and metal-[3,4] and semiconductor-based[5,6] nanostructures, the deviation of the optical conductivity from this Drude behavior can be observed experimentally. Hence, there has been a need to develop simple and tracy theoretical approach which can be applied to reproduce the experimental findings and, as a result, to obtain the sample and material parameters via fitting experimental data with theoretical formula. In 2001, Smith proposed an impulse response approach and Poisson statistics for evaluating the optical conductivity in the presence of memory or persistence of electron velocity effect.[7] Such a model is a classical generalization of the Drude formula and, therefore, has been named as Drude-Smith model. The complex optical conductivity given by the Drude-Smith model takes a form:

$$\sigma(\omega) = \frac{\sigma_0}{(1 - i\omega\tau)}\left[1 + \sum_{n=1}^{\infty}\frac{c_n}{(1 - i\omega\tau)^n}\right], \qquad (1)$$

where the coefficient $c_n = [0, -1]$ represents the fraction of the electron's original velocity that is retained after the nth electronic scattering event. Due to the presence of c_n terms in the formula, the Drude-Smith model can be used to investigate the deviation behavior of optical conductivity from the standard Drude formula. Interestingly, the coefficient c_n can be related to important electronic effects such as the persistence, backscattering, and even the localization. Since its publication in 2001, the Drude-Smith formula has been applied to examine optical conductivity in poor conductors and material systems near the metal-insulator transition,[8,9] the Anderson localization effects,[10] etc. In particular, the Drude-Smith formula has become a powerful theoretical tool in reproducing experimental results obtained from terahertz (10^{12} Hz or THz) time-domain spectroscopy (TDS) in recent years.[11] Nowadays, the THz TDS techniques have been widely applied to study newly developed condensed matter materials such as graphene,[12–14] high T_c superconductors,[15,16] strongly correlated systems,[17,18] metal nano-array structures,[19,20] to mention but a few. Using THz TDS techniques, it is possible to measure the real and imaginary parts of the optical conductivity for a material. Thus, through fitting the experimental results with the Drude-Smith formula, one can obtaine the important sample and material parameters such as the electron mobility (or relaxation time τ),[21] the electron density (or ratio between electron density and effective mass n_e/m^*),[22] and the information about electronic backscattering and localization (or c_n),[23,24] etc. As a

a)Electronic mail: wenxu_issp@aliyun.com

consequence, one can obtain these sample and material parameters optically without making Ohmic contacts and applying external magnetic field.

At present, the THz TDS techniques have been integrated with the high-magnetic field and low-temperature facilities to measure THz magneto-optical properties of a material.[25,26] Many important and interesting magneto-optical features, such as cyclotron resonance,[27,28] Faraday rotation,[29,30] optically detected quantum Hall effect,[31,32] and magneto-optical Kerr effect,[33,34] have been observed and investigated experimentally in THz bandwidth for different kinds of materials. The THz TDS based magneto-optical measurements can be used to obtain the real and imaginary parts of the longitudinal and transverse conductivities. By fitting these four experimental results with theoretical formula, we are able to obtain at least four independent physical properties. Thus, more information about sample and material parameters for a material or device can be determined via THz TDS based mangeto-optical measurements. On the other hand, one can use more experimental findings to examine the rightness of a theoretical model. From a fact that in poor metals[1,2] and in metal-[3,4] and semiconductor-based[5,6] nanostructures, the deviations of the optical conductivity from standard Drude behavior have been observed experimentally at zero magnetic field, and one would expect that the real and imaginary parts of longitudinal and transverse magneto-optical conductivities for these materials may differ from the standard Drude formula obtained in the presence of a magnetic field. At present, there is a lack of simple and tractable theoretical approaches to depict the magneto-optical conductivities which do not obey the standard Drude formula. Therefore, it is significant and important to develop a simple theoretical approach which can be easily applied to understand and reproduce experimental findings. In this work, we intend generalizing the Drude-Smith model proposed in the absence of a magnetic field to the case where the radiation and magnetic fields are applied simultaneously in the Faraday geometry. The prime motivation of the present study is to develop and provide a simple theoretical tool which can be easily applied to fit experimental data and to obtain the sample and material parameters via fitting.

The paper is organized as follows. We present the derivation of the Drude-Smith formula for the case where an external magnetic field is present in Section II. In Section III, the results are presented and discussed further. To see more application of the model in conjunction with experimental measurement, the theoretical results for the elements of the dielectric function matrix are presented and discussed. The main concluding remarks are summarized in Section IV.

II. THEORETICAL APPROACH

In the present study, we consider the situation where the external magnetic field is not high enough and the temperature is not low enough so that the effects induced by the Landau quantization can be neglected. In such a case, the classical theoretical approach can be used to study the magneto-optical properties of an electron gas system.

A. Drude-Smith model

In the absence of an external magnetic field, the current response function for an electron gas system can be written as:[7] $j(t) = j(0)e^{-t/\tau}$, which implies that the initial current $j(0) = n_e e^2/m^*$ decays exponentially to its equilibrium value with a relaxation time τ. The complex form of frequency-dependent optical conductivity is then obtained by the Fourier transformation of the current response function, which reads

$$\sigma(\omega) = \int_0^\infty j(t)e^{i\omega t}dt. \tag{2}$$

After considering that an electron in a material may suffer the first and subsequent collisions due to backscattering mechanism, Smith proposed[7] a modified current response function with a Poisson distribution

$$j(t) = j(0)e^{-t/\tau}\left[1 + \sum_{n=1}^{\infty}\frac{c_n}{n!}\left(\frac{t}{\tau}\right)^n\right], \tag{3}$$

where c_n is a parameter describing the persistence of the change of initial velocity for an electron after the nth scattering event. Hence, this model has taken into consideration that an electron experiences collisions which are randomly distributed in time but with an average time interval τ between collision events. Consequently, the complex optical conductivity in the absence of a magnetic field is obtained as Eq. (1).

B. Magneto-optical conductivity tensors

In the present study, we consider a situation where the radiation field is applied along the z-axis of the sample and is polarized linearly along the x-direction with an electric field strength E_x^i. The static magnetic field B is applied along the z-axis of the sample as well. Thus, the system is in Faraday geometry. The schematic illustration of the configuration for magneto-optical transmission measurement in Faraday geometry is shown in Fig. 1. As we know, in Faraday geometry, the effect of the cyclotron resonance can be observed due to coupling between magnetic and radiation fields. In such a case, because of the presence of the Lorentz force induced by the magnetic field, the electrons in an electronic material are in cyclotron orbit moving with a

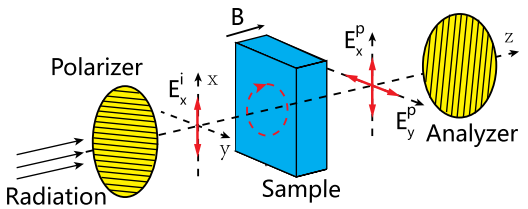

FIG. 1. The schematic illustration of the configuration for magneto-optical measurement in Faraday geometry. Here, the case for transmission experiment is shown. E_x^i is the electric field strength of the incident light which is polarized linearly along the x-axis via, e.g., a polarizer. The magnetic field B is applied along the z-axis. The electric field strengths of the light wave transmitted through the sample along different directions, E_x^p and E_y^p, are measured with the polarizer or analyzer.

frequency $\omega_c = eB/m^*$ called cyclotron frequency. As a result, the electric field of the light wave transmitted through the sample can be with E_x^p and E_y^p components, and the current response function has both longitudinal and transverse components, which are given as

$$j_x(t) = j(0)e^{-t/\tau}\cos(\omega_c t), \tag{4}$$

and

$$j_y(t) = -j(0)e^{-t/\tau}\sin(\omega_c t). \tag{5}$$

The corresponding longitudinal and transverse conductivities can be obtained through the Fourier transformation

$$\begin{aligned}\sigma_{xx}(\omega) &= j(0)\int_0^\infty e^{-t/\tau}\cos(\omega_c t)e^{i\omega t}dt \\ &= \frac{\sigma_0(1-i\omega\tau)}{(1-i\omega\tau)^2+(\omega_c\tau)^2},\end{aligned} \tag{6}$$

and

$$\begin{aligned}\sigma_{xy}(\omega) &= -j(0)\int_0^\infty e^{-t/\tau}\sin(\omega_c t)e^{i\omega t}dt \\ &= -\frac{\sigma_0\omega_c\tau}{(1-i\omega\tau)^2+(\omega_c\tau)^2},\end{aligned} \tag{7}$$

with $\sigma_0 = n_e e^2\tau/m^*$ being the conductivity in the absence of magnetic and radiation fields. Eqs. (6) and (7) are the same as the classical results for the Drude formula obtained in the presence of radiation and magnetic fields (see Appendix A).

Now, we take into account of the electron motion after the first and subsequent collisions. Using the Poisson statistics proposed by Smith, we have

$$j_x(t) = j(0)e^{-t/\tau}\cos(\omega_c t)\left[1+\sum_{n=1}^{\infty}\frac{a_n}{n!}\left(\frac{t}{\tau}\right)^n\right], \tag{8}$$

and

$$j_y(t) = -j(0)e^{-t/\tau}\sin(\omega_c t)\left[1+\sum_{n=1}^{\infty}\frac{a_n}{n!}\left(\frac{t}{\tau}\right)^n\right], \tag{9}$$

with a_n being a parameter describing the persistence of the change of initial velocity for an electron after the nth scattering event in the presence of a magnetic field. After Fourier transformations, the longitudinal and transverse magneto-optical conductivities now, respectively, become

$$\begin{aligned}\sigma_{xx}(\omega) &= \frac{j(0)}{2}\int_0^\infty e^{-t/\tau}\left(e^{i\omega_c t}+e^{-i\omega_c t}\right)\left[1+\sum_{n=1}^{\infty}\frac{a_n}{n!}\left(\frac{t}{\tau}\right)^n\right]e^{i\omega t}dt \\ &= \frac{\sigma_0/2}{1-i(\omega_c+\omega)\tau}\left[1+\sum_{n=1}^{\infty}\frac{a_n}{[1-i(\omega_c+\omega)\tau]^n}\right] \\ &\quad + \frac{\sigma_0/2}{1+i(\omega_c-\omega)\tau}\left[1+\sum_{n=1}^{\infty}\frac{a_n}{[1+i(\omega_c-\omega)\tau]^n}\right],\end{aligned} \tag{10}$$

and

$$\begin{aligned}\sigma_{xy}(\omega) &= -\frac{j(0)}{2i}\int_0^\infty e^{-t/\tau}\left(e^{i\omega_c t}-e^{-i\omega_c t}\right)\left[1+\sum_{n=1}^{\infty}\frac{a_n}{n!}\left(\frac{t}{\tau}\right)^n\right]e^{i\omega t}dt \\ &= \frac{i\sigma_0/2}{1-i(\omega_c+\omega)\tau}\left[1+\sum_{n=1}^{\infty}\frac{a_n}{[1-i(\omega_c+\omega)\tau]^n}\right] \\ &\quad - \frac{i\sigma_0/2}{1+i(\omega_c-\omega)\tau}\left[1+\sum_{n=1}^{\infty}\frac{a_n}{[1+i(\omega_c-\omega)\tau]^n}\right],\end{aligned} \tag{11}$$

both with a correction term with the coefficients a_n. In the above derivation, the recursion formulas for integration are given in Appendix B.

C. Special cases

Now, we discuss the special cases of the Drude-Smith formula obtained in the presence of the magnetic field. Firstly, when the external magnetic field is absent, we have $\sigma_{xy}(\omega) = 0$ and

$$\sigma_{xx}(\omega) = \frac{\sigma_0}{(1-i\omega\tau)}\left[1+\sum_{n=1}^{\infty}\frac{a_n}{(1-i\omega\tau)^n}\right], \tag{12}$$

which is the previous Drude-Smith formula obtained for $B = 0$.[7]

Secondly, when the radiation field becomes the DC field, i.e., $\omega = 0$, Eqs. (10) and (11) read, respectively

$$\begin{aligned}\sigma_{xx}(0) &= \frac{\sigma_0}{1+(\omega_c\tau)^2} + \frac{\sigma_0}{2}\sum_{n=1}^{\infty}a_n \\ &\quad \times\left[\frac{1}{(1-i\omega_c\tau)^{n+1}}+\frac{1}{(1+i\omega_c\tau)^{n+1}}\right],\end{aligned} \tag{13}$$

and

$$\begin{aligned}\sigma_{xy}(0) &= -\frac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2} + \frac{i\sigma_0}{2}\sum_{n=1}^{\infty}a_n \\ &\quad \times\left[\frac{1}{(1-i\omega_c\tau)^{n+1}}-\frac{1}{(1+i\omega_c\tau)^{n+1}}\right].\end{aligned} \tag{14}$$

When $a_n = 0$, $\sigma_{xx}(0) = \sigma_0/[1+(\omega_c\tau)^2]$ and $\sigma_{xy}(0) = -\sigma_0\omega_c\tau/[1+(\omega_c\tau)^2]$ so that the magneto-resistivities: $\rho_{xx} = 1/\sigma_0$ and $\rho_{xy} = B/n_e e$, which are the well-known classical results for magneto-resistivity tensors when quantum effects are not taken into consideration. We note that when $\omega = 0$ and $a_n = 0$, the magneto-conductivity or resistivity tensors are with only the real part. However, as long as there is at least one nonzero a_n, both $\sigma_{xx}(0)$ and $\sigma_{xy}(0)$ are complex quantities. It is known that the real part of the conductivity corresponds to the energy consuming process, namely, a process that an electron gains the energy from the external field and losses it due to electronic scattering mechanisms. Whereas the imaginary part of the conductivity describes the energy exchange process between electrons and external field applied on electrons, which does not change the energy

of the electron-field system. Such an effect is a consequence that the initial current decays exponentially to its equilibrium value with a relaxation time τ due to subsequent electronic collision events. This is electrically equivalent to the quasi-harmonic oscillation of electron motion.

Lastly, in Eqs. (10) and (11), the value of the coefficient a_n should be in-between 0 and -1. When $a_n = 0$, the Poisson statistics does not take into account and the magneto-optical conductivity tensors become the standard Drude formula (see Appendix A). $a_n = -1$ corresponds to a case of full backscattering mechanism to an electron, and the Poisson statistics has the strongest influence on magneto-optical conductivity tensors. For practical usage of the Drude-Smith formula, especially in fitting experimental data with the formula, one often first examines the results obtained by taking only a_1 term into consideration. If a_1 is in-between 0 and -1 and the good comparison between experimental and theoretical results can be achieved, $n > 1$ terms can be safely neglected. When a good fitting between experimental and theoretical results cannot be achieved by taking only $a_1 = [0, -1]$, higher order terms have to be included. Therefore, it is worthy and significant to discuss the case where the persistence of electron motion is retained for only one collision event (i.e., only the a_1 related term is considered). When $a_n = 0$ for $n > 1$, after defining $a_1 = a$, the longitudinal and transverse current response functions become, respectively

$$j_x(t) = j(0)e^{-t/\tau}\cos(\omega_c t)\left(1 + \frac{at}{\tau}\right), \tag{15}$$

and

$$j_y(t) = -j(0)e^{-t/\tau}\sin(\omega_c t)\left(1 + \frac{at}{\tau}\right), \tag{16}$$

and the longitudinal and transverse magneto-optical conductivities are given, respectively, as

$$\sigma_{xx}(\omega) = \sigma_0 \frac{1 - i\omega\tau}{(1 - i\omega\tau)^2 + (\omega_c\tau)^2} + \frac{a\sigma_0}{2}\left[\frac{1}{[1 - i(\omega + \omega_c)\tau]^2} + \frac{1}{[1 - i(\omega - \omega_c)\tau]^2}\right], \tag{17}$$

and

$$\sigma_{xy}(\omega) = -\sigma_0 \frac{\omega_c\tau}{(1 - i\omega\tau)^2 + (\omega_c\tau)^2} + \frac{ia\sigma_0}{2}\left[\frac{1}{[1 - i(\omega + \omega_c)\tau]^2} - \frac{1}{[1 - i(\omega - \omega_c)\tau]^2}\right]. \tag{18}$$

Here, both $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$ are complex quantities, and the cyclotron resonance effect can be seen when $\omega \sim \omega_c$.

III. RESULTS AND DISCUSSIONS

First of all, it is necessary and interesting to look into how a magnetic field and the Poisson statistics affect the impulsive current response function that is the basis of the Drude-Smith model. To see the effects more clearly, we consider the situation where $a_n = 0$ for $n > 1$ and use Eqs. (15) and (16) for examination. In Fig. 2, we show the time dependence of the longitudinal [$j_x(t)$ in (a)] and transverse [$j_y(t)$ in (b)] impulsive current response functions at a fixed relaxation time τ for $B = 0$ and $B = 5\,\mathrm{T}$ and for different coefficients a. For sake of demonstration only, here we take the electron effective mass to be $m^* = 0.065m_e$ (for semiconductor such as GaAs) with m_e being the rest electron mass and the relaxation time to be $\tau = 0.5$ ps (for typical momentum relaxation time in semiconductors). For a case of $B = 0$ and $a = 0$ (the black curves in Fig. 2), which corresponds to the standard Drude formula at $B = 0$, $j_y(t) = 0$ and $j_x(t)$ are always positive and decay exponentially with t. When $B \neq 0$ and $a = 0$ (red curves in Fig. 2), which corresponds to the standard Drude formula for $B \neq 0$, both $j_x(t)$ and $j_y(t)$ oscillate with a frequency $\omega_c = eB/m^*$ and decay exponentially with t purely. When $B \neq 0$ and $a = (0, -1]$ (e.g., for $a = -0.5$ and -1 in Fig. 1), both $j_x(t)$ and $j_y(t)$ still oscillate with a frequency ω_c, but their decay behavior is modified markedly by the presence of the $1 + at/\tau$ term in Eqs. (15) and (16). From Fig. 2, we see that the presence of the magnetic field can induce the oscillations of the current response functions with a frequency ω_c, which can lead to a change of sign for the current response functions. Moreover, the usage of the Poisson statistics in Drude-Smith model [i.e., the presence of $1 + at/\tau$ term in Eqs. (15) and (16)] can change the feature of the time decay of the longitudinal and transverse current response functions.

Now, we discuss the basic features of the longitudinal and transverse magneto-optical conductivities given by Drude-Smith model. Again, to see the effects more clearly, we consider the situation where $a_n = 0$ for $n > 1$ and use Eqs. (17) and (18) for investigation. We take $m^* = 0.065m_e$ and $\tau = 0.5$ ps in the calculation. In Fig. 3, we show the real and imaginary parts of the longitudinal and transverse magneto-optical conductivities as a function of radiation frequency $\nu = \omega/2\pi$ at a fixed relaxation time τ and a fixed magnetic field $B = 5\,\mathrm{T}$ for different coefficients a. When $B = 5\,\mathrm{T}$ and $m^* = 0.065m_e$, $\omega_c/2\pi = 2.16\,\mathrm{THz}$. For $\tau = 0.5$ ps, $\omega_c\tau \sim 1$

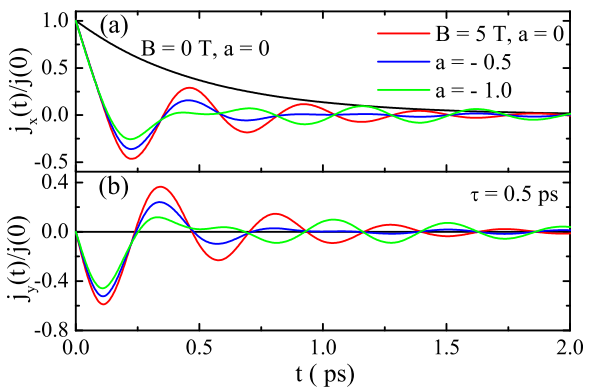

FIG. 2. The time dependence of the longitudinal [$j_x(t)$ in (a)] and transverse [$j_y(t)$ in (b)] impulsive current response functions at a fixed relaxation time $\tau = 0.5$ ps for $B = 0$ (black curves) and $B = 5\,\mathrm{T}$ and for different coefficients a as indicated. Here, $j(0) = n_e e^2/m^*$.

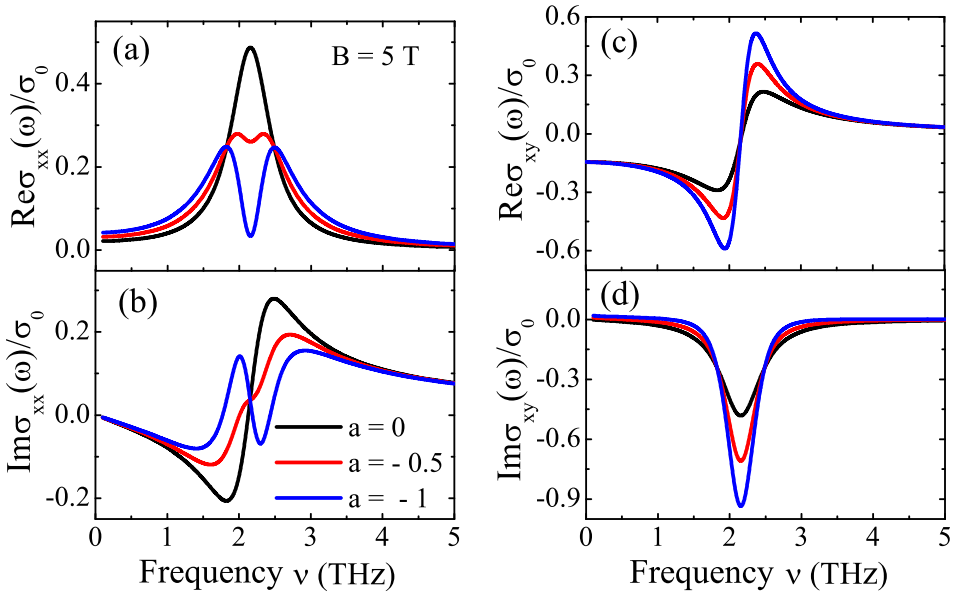

FIG. 3. The real and imaginary parts of the longitudinal [in (a) and (b), respectively] and transverse [in (c) and (d), respectively] magneto-optical conductivities as a function of radiation frequency $\nu = \omega/2\pi$ at a fixed relaxation time $\tau = 0.5$ ps and a magnetic field $B = 5$ T for different coefficients a as indicated. Here, $\sigma_0 = n_e e^2\tau/m^*$.

can be satisfied. Thus, the cyclotron resonance effect can be observed in $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$ in THz regime where $\omega\tau \sim 1$. As we know, the real part of optical conductivity corresponds to the process that an electron gains the energy from the external field and losses it due to electronic scattering mechanisms. The peak for Re $\sigma_{xx}(\omega)$ in Fig. 3(a) at $a = 0$ is induced by the resonant absorption due to the cyclotron resonance effect. However, in the presence of the backscattering or localization mechanism (i.e., for a nonzero coefficient a), the absorption peak splits into two peaks and the peak position at $a = 0$ becomes the valley position at $a \neq 0$ as shown in Fig. 3(a) for $a = -0.5$ and $a = -1$. With decreasing the value of the coefficient a (note that a should be a negative value), the effect of backscattering becomes stronger and the splitting of the peak for cyclotron resonance becomes more obvious. From Fig. 3(c), we see that the presence of a nonzero coefficient a does not change the basic features of Re$\sigma_{xy}(\omega)$. However, a stronger backscattering effect or a smaller a can lead to a higher peak and a deeper valley in Re$\sigma_{xy}(\omega)$. The imaginary part of the optical conductivity describes the energy exchange process between electrons and applied external field. The results shown in Fig. 3(b) indicate that the presence of the backscattering mechanism can change the features of Im$\sigma_{xx}(\omega)$ significantly. With varying the coefficient a, Im$\sigma_{xx}(\omega)$ at $a = 0$ and $a = -1$ depends very differently on frequency $\nu = \omega/2\pi$ around the cyclotron frequency $\omega \sim \omega_c$. In contrast, we find that the presence of a nonzero coefficient a does not change the basic features of Im$\sigma_{xy}(\omega)$ which is always negative. However, with decreasing the coefficient a, the valley in Im$\sigma_{xy}(\omega)$ around the cyclotron frequency $\omega \sim \omega_c$ looks deeper and the position of the valley does not shift with varying a.

It should be noted that once the magneto-optical conductivity tensor is obtained, the components of the dielectric function matrix can be evaluated simply through[17]

$$\varepsilon_{xx}(\omega) = \varepsilon_b + \frac{i\sigma_{xx}(\omega)}{\varepsilon_0\omega}, \tag{19}$$

and

$$\varepsilon_{xy}(\omega) = \frac{i\sigma_{xy}(\omega)}{\varepsilon_0\omega}, \tag{20}$$

where ε_b is the dielectric constant of the host material and ε_0 is the permittivity in the vacuum. They are, in general, the complex quantities.

In 2004, Ino and co-workers measured the real and imaginary parts of $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$ for InAs[34] via THz Kerr effect experiments by using THz TDS technique at room temperature. In general, the transmission measurement by the THz TDS technique can be obtained directly the real and imaginary parts of optical conductivity.[18] On the other hand, the THz TDS based reflection experiments can normally be applied to determine the real and imaginary parts of the dielectric response function or dielectric constant[34] via measuring the strength and phase of the THz waves reflected from the sample. In the paper by Ino and co-workers,[34] they took the standard Drude formula for $B \neq 0$ to fit the experimental data and obtained the sample and material parameters such as the electronic relaxation time and electron density in InAs samples. We have applied the present Drude-Smith formula to fit the experimental data for InAs[34,35] and have found the better agreement between experimental and theoretical results. However, we know that generally the electronic localization effect in bulk InAs is considered to be very weak. Therefore, in the present study, we calculate the real and imaginary parts of $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$ for nano-granular VO_2 films in which the electronic metal and insulator phases can co-exist[9] and the electronic localization effect can be markedly observable. The metal-insulator transition in nano-granular VO_2 films has been studied by using THz TDS techniques in the absence of a magnetic field.[9,36] When $B = 0$, the Drude-Smith formula can fit very well with the experimental results obtained for nano-granular VO_2 films and the corresponding value of $a = -0.66$ can be determined.[9] In Fig. 4, we demonstrate how the parameter a in Eqs. (19) and (20) affects the real and imaginary parts of $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$ for nano-granular VO_2 films in the presence of a magnetic field B. In the calculation, we take $a = -0.66$ determined

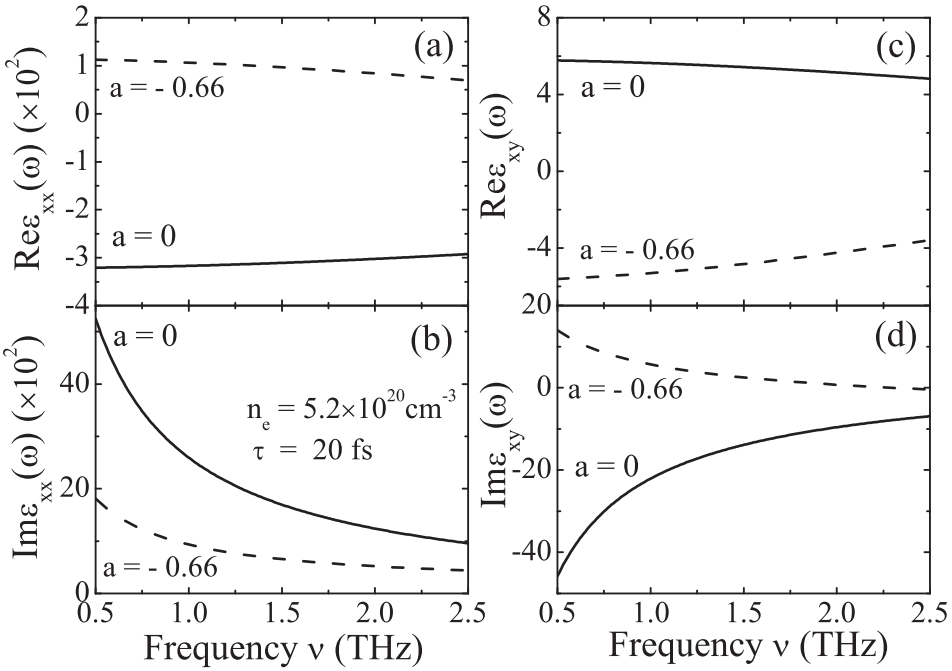

FIG. 4. The real and imaginary parts of dielectric tensors as a function of radiation frequency $\nu = \omega/2\pi$ at a fixed magnetic field strength $B = 5\,\mathrm{T}$ for different values of a. The solid and dashed curves are for $a = 0$ and $a = -0.66$, respectively.

experimentally[9] at $B = 0$, $\varepsilon_b = 9.0$, the electron density[9] $n_e = 5.2 \times 10^{20}\,\mathrm{cm}^{-3}$, the relaxation time[9] $\tau = 20$ fs, and the electron effective mass[9] $m^* = 2.0m_e$. We find that by using these parameters, the value $\omega_c\tau$ is relatively small at $B = 5\,\mathrm{T}$ for VO_2 so that the effect of Landau quantization can be neglected and the cyclotron resonance cannot be observed in the THz frequencies. In Fig. 4, we plot the real and imaginary parts of $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$ as a function of radiation frequency for a nano-granular VO_2 film at a fixed magnetic field strength $B = 5\,\mathrm{T}$ for different values of $a = 0$ (solid curves) and $a = -0.66$ (dashed curves). As we can see from Figs. 4(a) and 4(c), because $\omega\tau$ is relatively small for VO_2 in THz regime, $\mathrm{Re}\varepsilon_{xx}(\omega)$ and $\mathrm{Re}\varepsilon_{xy}(\omega)$ depend weakly on ω for $a = 0$ and $a = -0.66$. Similar to the case of $B = 0$, a big difference of $\mathrm{Re}\varepsilon_{xx}(\omega)$ can be found for $a = 0$ and $a = -0.66$ at $B = 5\,\mathrm{T}$ in THz regime. Both $\mathrm{Re}\varepsilon_{xx}(\omega)$ and $\mathrm{Re}\varepsilon_{xy}(\omega)$ can change from the negative values at $a = 0$ to the positive values at $a = -0.66$ for $B = 5\,\mathrm{T}$. As shown in Figs. 4(b) and 4(d), both $\mathrm{Im}\varepsilon_{xx}(\omega)$ and $\mathrm{Im}\varepsilon_{xy}(\omega)$ depend rather strongly on radiation frequency ω and their values at $a = 0$ and $a = -0.66$ differ significantly at $B = 5\,\mathrm{T}$ for a nano-granular VO_2 film. Particularly, $\mathrm{Im}\varepsilon_{xy}(\omega)$ decreases with increasing ω at $a = -0.66$ and increases with ω at $a = 0$.

The results shown in Figs. 3 and 4 indicate that for an electronic system in which the electronic backscattering or localization effect is present, the parameter a in the Drude-Smith formula for magneto-optical conductivities can affect strongly the real and imaginary parts of conductivity tensors $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$ and dielectric function tensors $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$. In the presence of the magnetic field, the nonzero real and imaginary parts of $\sigma_{xy}(\omega)$ and $\varepsilon_{xy}(\omega)$ can be present. Therefore, the real and imaginary parts of conductivity tensors or dielectric function tensors determined from THz TDS based transmission or reflection experiments for a sample can be applied to fit the Drude-Smith formula and to determine the sample and material parameters via fitting experimental data with theoretical formula. The most significant effect of the electronic backscattering and localization on magneto-optical properties can be observed for $\mathrm{Re}\sigma_{xx}(\omega)$ where the peak of the cyclotron resonance splits into two peaks in the Drude-Smith model and the peak position given by standard Drude formula becomes the valley position. When the effect of the electronic backscattering and localization is strong enough, e.g., for nano-granular VO_2 films, $\mathrm{Im}\varepsilon_{xy}(\omega)$ decreases with increasing ω in sharp contrast to a fact that it increases with ω given by standard Drude formula.

IV. CONCLUDING REMARKS

In this work, we have developed a simple and tractable theoretical approach to evaluate the magneto-optical conductivity tensors in an electron gas system in Faraday geometry. Such an approach is a generalization of the optical conductivity derived from the impulse response approach and Poisson statistics proposed by Smith for a case where the external magnetic field is absent. We have examined the dependence of real and imaginary parts of conductivity tensors $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$ and dielectric function tensors $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$ on the radiation frequency, magnetic field strength, and parameter a in the Drude-Smith formula. The main conclusions obtained from this study are summarized as follows. (i) By applying the Drude-Smith model to derive the magneto-optical conductivity tensors for an electron gas system, we can obtain the real and imaginary parts of $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$ with the coefficients a_n induced by the Poisson statistics and related to electronic backscattering or localization effect. (ii) The Drude-Smith formula generalized in this study for $B \neq 0$ can be applied to study the magneto-optical properties such as the real and imaginary parts of conductivity tensors and dielectric function tensors in the electronic system. (iii) The presence of the a_n terms in the Drude-Smith

formula can change dramatically the features of the longitudinal and transverse magneto-optical conductivities and dielectric function elements. Thus, the formula can be applied to study the electronic materials and devices whose magneto-optical conductivities deviate from the standard Drude behavior and to obtain the information about electronic backscattering and localization which cannot be obtained directly from magneto-transport measurement. (iv) The most significant effect of the electronic backscattering and localization on magneto-optical properties can be observed for the real part of the longitudinal conductivity and for the imaginary of transverse dielectric function tensor. The peak of the cyclotron resonance in $\mathrm{Re}\sigma_{xx}(\omega)$ broadens and splits into two peaks in the Drude-Smith model for a nonzero parameter a and the peak position given by standard Drude formula for $a=0$ becomes the valley position. $\mathrm{Im}\varepsilon_{xy}(\omega)$ decreases with increasing ω in the Drude-Smith model, e.g., nano-granular VO_2 films, and increases with ω at $a=0$ in the standard Drude model.

At present, the techniques of the terahertz (THz) time-domain spectroscopy (TDS) have been applied to investigate the magneto-optical properties of the condensed matter materials and electronic devices in conjunction with the high magnetic field and low temperature facilities. Such techniques can be used to measure the real and imaginary parts of conductivity tensors $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$ and dielectric function tensors $\varepsilon_{xx}(\omega)$ and $\varepsilon_{xy}(\omega)$ and, therefore, have been applied to study important and interesting magneto-optical effects such as the cyclotron resonance, Faraday rotation, quantum Hall effect, Kerr effect, etc. In recent years, the Drude-Smith formula for $B=0$ has been widely adopted in the theoretical analysis of the experimental findings from the THz TDS measurements. On this basis, we believe that the magneto-optical conductivity tensors obtained from the Drude-Smith model for $B\neq 0$ can be used appropriately to reproduce the experimental findings and to fit with experimental data in order to determine the important sample and material parameters.

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APPENDIX A: THE CLASSICAL DRUDE FORMULA FOR $B\neq 0$

For a case where the light field is polarized linearly along the x-direction and the magnetic field is applied along the z-direction, we have the electric field strength of the radiation field as $\mathbf{E}_x^i=(E_x,0,0)e^{-i\omega t}$ and the static magnetic field strength as $\mathbf{B}=(0,\ 0,\ B)$, with ω being the radiation frequency. For linear response of an electron in an electron gas system to the light field, the drift velocity of the electron oscillates with the optical field synchronously and $\mathbf{v}=(v_x,v_y,0)e^{-i\omega t}$ due to cyclotron movement induced by the presence of the magnetic field. Thus, the equation of motion for the electron is given by

$$\frac{dv_x}{dt}=-\frac{e}{m^*}(E_x+v_yB)-\frac{v_x}{\tau}, \tag{A1}$$

and

$$\frac{dv_y}{dt}=\frac{e}{m^*}(v_xB)-\frac{v_y}{\tau}, \tag{A2}$$

with τ being the electronic relaxation time. By solving the above equations, we get the steady-state electron velocities as

$$v_x=-\frac{eE_x\tau}{m^*}\frac{1-i\omega\tau}{(1-i\omega\tau)^2+(\omega_c\tau)^2}, \tag{A3}$$

and

$$v_y=\frac{\omega_c\tau}{1-i\omega\tau}v_x, \tag{A4}$$

where $\omega_c=eB/m^*$. By definition, the current density $\mathbf{j}=-n_e e\mathbf{v}$ with n_e being the electron density. Taking $\sigma_0=n_e e^2\tau/m^*$, the components of the current density are

$$j_x=\sigma_0\frac{1-i\omega\tau}{(1-i\omega\tau)^2+(\omega_c\tau)^2}E_x, \tag{A5}$$

and

$$j_y=\sigma_0\frac{\omega_c\tau}{(1-i\omega\tau)^2+(\omega_c\tau)^2}E_x. \tag{A6}$$

By definition, the matrix $\mathbf{j}=\sigma\cdot\mathbf{E}$ with σ being the conductance matrix. Thus, we obtain the longitudinal and transverse conductivities as

$$\sigma_{xx}=\frac{1-i\omega\tau}{(1-i\omega\tau)^2+(\omega_c\tau)^2}\sigma_0, \tag{A7}$$

and

$$\sigma_{xy}=-\frac{\omega_c\tau}{(1-i\omega\tau)^2+(\omega_c\tau)^2}\sigma_0. \tag{A8}$$

APPENDIX B: RECURSION FORMULAS

To do the Fourier transformation in this work, the following formulas have been applied to carry out the integrations.

$$\int_0^\infty e^{-t/\tau}e^{\pm i\omega_c t}e^{i\omega t}(t/\tau)^n dt = \frac{n}{1\mp i(\omega_c\pm\omega)\tau}\int_0^\infty e^{-t/\tau}e^{\pm i\omega_c t}e^{i\omega t}(t/\tau)^{n-1}dt, \tag{B1}$$

$$\int_0^\infty e^{-t/\tau}e^{\pm i\omega_c t}e^{i\omega t}(t/\tau)^n dt=\frac{n!\tau}{[1\mp i(\omega_c\pm\omega)\tau]^{n+1}}, \quad (n\geq 0). \tag{B2}$$

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