# Interpretation of the morphology and crystallography of precipitate $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ in $\mathrm{L1}_{0}-\mathrm{TiAl}(\mathrm{Nb})$-based intermetallics by invariant line theory 

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#### Abstract

The coherent precipitate $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ in the Nb-modified TiAl-based intermetallic compound with a nominal composition of $\mathrm{Ti}-48$ at $\%$ $\mathrm{Al}-10 \mathrm{at} \% \mathrm{Nb}$ was investigated by transmission electron microscopy. The crystallographic and morphologic characteristics of the precipitating phase have been calculated based on invariant line theory. It is revealed that in the $\gamma-\mathrm{TiAl}(\mathrm{Nb}) / \gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ couples, both the morphological and crystallographical characteristics of the precipitate $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ could be predicted by the three-dimensional phase transformation invariant line model. The needle direction of [001] at the early stage for coherent precipitate $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ and [111] for equilibrium phase $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ have both been explained well. The characterization of morphology and crystallography of the precipitate reaction in $\gamma-\mathrm{TiAl}_{1} / \gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ system sustain the postulate that precipitates are bounded by unrotated planes (eigenplanes) when three real eigenvalues exist.


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## 1. Introduction

Titanium aluminide intermetallic compounds are candidates as high temperature structural materials due to their high specific strength, high melting point and good oxidation resistance at elevated temperature. A lot of research efforts were devoted to this kind of alloys [1-3].
$\gamma$ - TiAl alloys containing $5-10 \mathrm{at} \% \mathrm{Nb}$ are regarded as the most perspective high temperature intermetallics. Precipitation behaviors of carbides and silicides in $\mathrm{C}+\mathrm{Si}$-alloyed $\gamma-\mathrm{TiAl}$ $[4,5]$ and precipitation behavior of carbides in Ag -modified $\mathrm{L1}_{2}-\mathrm{Al}_{3} \mathrm{Ti}$ and $\mathrm{L1}_{0}-\mathrm{TiAl}(\mathrm{Ag})[6,7]$ have been investigated previously. However, to the knowledge of the present authors,

[^0]precipitation behavior of Nb in high Nb -containing $\gamma$ - TiAl alloys has not been studied in detail. Hellwig et al. [8] and Chen et al. [9] have reported the diagram of the $1273 \mathrm{~K}, 1473 \mathrm{~K}$ and 1673 K isothermal sections of the $\mathrm{Ti}-\mathrm{Al}-\mathrm{Nb}$ system. Crystallographic structure of the phase $\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}\left(\gamma_{1}\right)$ equilibrium phase was then reported by Chen et al. [10]. The $\mathrm{L} 1_{0}-\mathrm{TiAl}$ phase is of an ordered face-centered tetragonal structure with parameters $a=0.3976 \mathrm{~nm}$ and $c=0.4049 \mathrm{~nm}$. The $\gamma_{1^{-}}$ $\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ phase is tetragonal with lattice parameter $a$ in the range $0.558-0.584 \mathrm{~nm}$ and $c$ in the range $0.815-0.845 \mathrm{~nm}$. Its space group is $P 4 / \mathrm{mmm}$. The equilibrium morphology takes needle-like shape with an axis of $\langle 111]$ [11].

We have reported that the $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ could also precipitate as metastable phase from the $\gamma$-TiAl matrix [12]. A detailed transmission electron microscopy (TEM) study was done on the morphological and crystallographical characteristics of $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ precipitation in the $\gamma$ - TiAl matrix. The
$\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ precipitates, which formed after a normal quench at 1473 K and ageing treatment at 1073 K , were needle-like with a growing axis parallel to [001] direction of the matrix.

The orientation relationship between the $\gamma_{1}$ phase and the $\gamma$-TiAl matrix was $[001]_{\gamma_{1}} / /[001]_{\gamma},(100)_{\gamma_{1}} / /(110)_{\gamma}$ and $(010)_{\gamma_{1}} / /(\overline{1} 10)_{\gamma}$.

The crystallography and morphologies of products of phase transformations have been the subject of major research for many years. In recent decades, many rival models have been developed to account for orientation relationship (OR), habit plane (HP) and growing orientation of the transforming products. It notably includes phenomenological theory of martensite crystallography [13,14], invariant line strain model [1517], structural ledges [18], O-lattice [19], $\Delta g$ parallelism rules [20], edge-to-edge matching [21,22] and planar interphase boundaries migrating mechanism [23], etc. They have been successfully applied to a wide range of transformation products. However, none of them seem to be sufficiently general. Invariant line theory is proved to be effective to those diffu-sion-controlled phase transformations whose products have a shape of needle, short-bar, platelet and facet, etc [24-26].

The aim of this paper is to make a rational explanation for the morphology and crystallography of $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ precipitate at the early stage in a $\mathrm{L1}_{0}$-TiAl matrix and for those of $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ equilibrium phase in $\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ alloy with three-dimensional invariant line theory. The simple and distinct calculation method of three-dimensional invariant line used here has been reported elsewhere [27], which could allow one to validate completely our calculation process. The needle direction of [001] at early stage for coherent precipitate $\gamma_{1^{-}}$ $\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ and [111] for equilibrium phase $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ will been explained based on the invariant line theory.

## 2. Experiments

The material used in this study has a nominal composition of $\mathrm{Ti}-48 \mathrm{Al}-10 \mathrm{Nb}$ (atom percentage). The alloy was prepared by levitation melting of high purity (99.9\%) $\mathrm{Ti}, \mathrm{Al}$ and Nb in copper crucible under argon atmosphere. Al and Nb are added as interalloy $\mathrm{AlNb}_{3}$. Heart-button ingot of about 200 g was reversed and remelted for three times in order to reduce segregation. Since the weight loss of this melting method is very low, the final alloy composition verified by X-ray Fluorescence spectrometer (XRF) is close to the nominal composition (see Table 1). Then, the alloy ingot was homogenized at 1473 K for 4 h and ageing at 1073 K for 34 h .

The TEM samples were cut into thin slices from the ingots followed by mechanical grinding to about $60 \mu \mathrm{~m}$ thickness. Preparation of discs was carried out using double-jet electropolishing technique with an electrolyte consisting of $65 \mathrm{vol} \%$

Table 1
The nominal and analytical composition of the $\mathrm{Ti}-48 \mathrm{Al}-10 \mathrm{Nb}$ alloy

|  | Ti | Al | Nb | Fe | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Nominal | 42 | 48 | 10 | 0 | 100 |
| Analytical | 43.28 | 45.10 | 10.71 | 0.81 | 100 |

methanol, $30 \mathrm{vol} \%$ butanol and $5 \mathrm{vol} \%$ perchloric acid at 30 V and 253 K to 243 K . The foils were examined in a Philips Tecnai F20 transmission electron microscope. High-resolution TEM (HRTEM) investigation was also performed.

## 3. Results and discussion

### 3.1. Microstructure and crystallographic characteristics of the Ti-48Al-10Nb alloy

Fig. 1 shows the microstructure of the $\mathrm{Ti}-48 \mathrm{Al}-10 \mathrm{Nb}$ alloy taken at $[110]_{\gamma}$ (a) and $[001]_{\gamma}$ (c). Fig. 1(b), (c), (e) and (f) are selected area electron diffractions taken down [110]m, [010]m, [001] m , and [013]m, respectively. It is noticed that the diffraction pattern of $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ does actually overlap on that of $\gamma$ TiAl shown in Fig. 1(b) taken down $[110]_{\gamma} / /[100]_{\gamma_{1}}$. This could be revealed by the stereographic projections corresponding to plane (Fig. 1(g)) or corresponding to direction (Fig. 1(h)). It could also be verified by calculating interplanar distances based on the crystallographic parameters of the $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ and $\gamma$ - TiAl couple. The Bain orientation relationship between $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ and $\mathrm{L1}_{0}-\mathrm{TiAl}(\mathrm{Nb})$ is clearly illustrated by such stereographic projections, i.e., $[001]_{\gamma_{1}}$ $/ /[001]_{\gamma},(100)_{\gamma_{1}} / /(110)_{\gamma}$ and $(010)_{\gamma_{1}} / /(\overline{1} 10)_{\gamma}$.

The microstructure of the alloy consists of matrix and needlelike precipitates. The EDP and composition analysis as we have done in our previous study [12] confirmed that the matrix is $\gamma$ $\mathrm{TiAl}(\mathrm{Nb})$ with $\mathrm{L1}_{0}$ structure and the precipitates are $\gamma_{1^{-}}$ $\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ with orthorhombic structure. The $\gamma_{1}-\mathrm{TiAl}_{3} \mathrm{Nb}_{9}$ precipitates (denoted as " p ") nucleate in the $\mathrm{L1}_{0}$ matrix (denoted as " $m$ ") homogeneously adopting a needle-like shape with growing orientation $[001]_{\gamma_{1}} / /[001]_{\gamma}$, a habit plane of $(110)_{\mathrm{m}}$ and a Bain orientation relationship $\left([001]_{\gamma_{1}} / /[001]_{\gamma}\right.$, $\left.(100)_{\gamma_{1}} / /(110)_{\gamma} \operatorname{and}(010)_{\gamma_{1}} / /(\overline{1} 10)_{\gamma}\right)$ with the matrix. Fig. 2 is a high-resolution TEM (HRTEM) image showing the lattice corresponding between the precipitate and the matrix viewed at $[010]_{\gamma}$. Inset is an inverse fast Fourier transition (IFFT) image down $[001]_{\gamma}$, from which the cross-section of the precipitate phase is revealed to take as quadratic form and the precipitate was bounded by planes $(100)_{\gamma_{1}} / /(110)_{\gamma}$ and $(010)_{\gamma_{1}} / /(\overline{1} 10)_{\gamma}$ which indicated the precipitate took a parallelepiped shape. Notice the IFFT image was taken from our previous work [12] for further discussing the morphology. The original HRTEM image is corresponding to Fig. 7 in literature [12].

### 3.2. Morphology and crystallography of the $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ precipitating phase

### 3.2.1. Lattice transformation and deformation

As mentioned above, invariant line theory is proved to be effective to those diffusion-controlled phase transformations whose products have a shape of needle, short-bar, platelet and facet, etc. For a phase transformation system of $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ (precipitate, tetragonal ( T$)$ )/ $/ 1_{0}-\mathrm{TiAl}(\mathrm{Nb})$ (matrix, face-centered tetragonal (FCT)), the lattice correspondence can be regarded as type of FCT to body-centered tetragonal (BCT)


Fig. 1. The microstructures of the Ti-48 at $\% \mathrm{Al}-0 \mathrm{at} \% \mathrm{Nb}$ alloy taken at $[110]_{\mathrm{m}}$ (a) and [001] $\mathrm{m}_{\mathrm{m}}$ (d). Notice the needle-like projection morphology of precipitates in (a) and spot-like projection morphology of precipitates in (d); (b), (c), (e) and (f) are selected area electron diffractions taken down $[110]_{\mathrm{m}},[010]_{\mathrm{m}},[001]_{\mathrm{m}}$, and [013] m , respectively; ( g ) and (h) are stereographic projections of planes $(\mathrm{g})$ and of directions $(\mathrm{h})$ showing the orientation relationship between the matrix and the precipitate, pole center is $[001]_{\gamma} /[001]_{\gamma_{1}}$
based on the fact that the unit cell of the phase $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ contains eight small BCT cells [10] shown in Fig. 3(a). Crystal structure of the $\gamma$-TiAl matrix is shown in Fig. 3(b). Fig. 4 shows a simple lattice corresponding model of Bain strain of FCT/BCT system. The coordinate system is also denoted. The main strain
is denoted with $\eta_{\mathrm{i}}$ (the subscripts " m " and " p " mean matrix and precipitate, respectively). The generalized Bain deformation (the parameters takes $a_{\mathrm{m}}=0.3976 \mathrm{~nm}, c_{\mathrm{m}}=0.4051 \mathrm{~nm}$ for $\gamma-\mathrm{TiAl}$ and $a_{\mathrm{p}}=0.58 \mathrm{~nm}, c_{\mathrm{p}}=0.81 \mathrm{~nm}$ for $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ ) can be expressed as:


Fig. 2. The HRTEM image of the precipitate viewed at $[010]_{\gamma} / /[100]_{\gamma_{j}}$; in-set (a) shows an IFFT image of the precipitate viewed at $[001]_{\gamma} / /[001]_{\gamma_{1}}$.

$$
\begin{align*}
B & =\left(\begin{array}{ccc}
\eta_{1} & 0 & 0 \\
0 & \eta_{2} & 0 \\
0 & 0 & \eta_{3}
\end{array}\right)=\left(\begin{array}{ccc}
\frac{a_{\mathrm{p}} / 2}{a_{\mathrm{m}} / \sqrt{2}} & 0 & 0 \\
0 & \frac{a_{\mathrm{p}} / 2}{a_{\mathrm{m}} / \sqrt{2}} & 0 \\
0 & 0 & \frac{c_{\mathrm{p}} / 2}{c_{\mathrm{m}}}
\end{array}\right)  \tag{3}\\
& =\left(\begin{array}{ccc}
1.03149 & 0 & 0 \\
0 & 1.03149 & 0 \\
0 & 0 & 0.99975
\end{array}\right) \tag{1}
\end{align*}
$$



Larger cell

After lattice deformation, crystallographic orientation relationship (O.R.) between the matrix and the precipitate can be expressed as:

$$
\begin{align*}
& (001)_{\mathrm{m}} / /(001)_{\mathrm{p}} \\
& {[110]_{\mathrm{m}} / /[100]_{\mathrm{p}}}  \tag{2}\\
& {[110]_{\mathrm{m}} / /[100]_{\mathrm{p}}}
\end{align*}
$$

It is obviously that this orientation relationship is typical Bain relationship.

### 3.2.2. Rigid-body rotation

Based on the invariant line strain model, one or several rigid-body rotations of the new phase may occur with respect to the matrix in order to reach the best orientation relationship (or the best matching) after lattice deformation. In this study, the precipitating phase needs one step rotation to get the phase transformation invariant line.

This step is to let the lattice of precipitating phase to rotate an angle of $\theta$ around $X$ axis (the $X$ axis in the reference coordinate system), and the rotation matrix $R$ is easy to take out after Wayman's results [28]:
$R=\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta\end{array}\right)$
According to the determination of the invariant line strain, the calculated rotation angle is $0.18^{\circ}$. In this system, one step rotation is enough to realize invariant line strain. This will lead to an invariant line becoming reality and to define an orientation relationship between the precipitate and matrix.


Sub cell
O atom Ti

- atom Nb
- atom Al , one at body-center of large cell, others at body-center of eight sub cells.
(b)

$\bigcirc$ atom Ti
- atom Al ; All atoms lie in crystalline plane (002).

Fig. 3. Crystal structures of $\gamma 1-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ (a) and $\gamma-\mathrm{TiAl}$ (b).


Fig. 4. A schematic showing the lattice rearrangement of FCT-to-BCT transformation.

### 3.2.3. Calculation of invariant line

The phase transformation invariant line deformation $A$ is:
$A=R B=\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta\end{array}\right)\left(\begin{array}{ccc}\eta_{1} & 0 & 0 \\ 0 & \eta_{2} & 0 \\ 0 & 0 & \eta_{3}\end{array}\right)$
According to the definition of the invariant line, $(A-I) X=0$, the rotation angle $\theta$ can be calculated to be:
$\cos \theta=\frac{1+\eta_{3} \eta_{2}}{\eta_{3}+\eta_{2}}$
" $\theta$ " (calculated to be $0.18^{\circ}$ ) is the rigid-body rotation angle to get invariant line after lattice deformation and defines the last orientation relationship between matrix and the precipitate. That is to say, " $\theta$ " is the angle of departure between the last O.R. and O.R. (Eq. (2)).

By linear algebra calculation, it is easy to get the eigenvalues and eigenvectors of the deformation matrix $A$ [28]. For detailed calculation, see online Appendix A in Ref. [26]. All the results are shown in Table 2. The results show that one of eigenvalues always equals to 1 and corresponding eigenvector is the phase transformation invariant line. All the eigenvalues are real which make the precipitate possible to be bounded by three eigenplanes.

### 3.2.4. Prediction of the crystallography of the $F C T \rightarrow B C T$ precipitation

Based on the calculation above, the crystallographic characteristics of the FCT $\rightarrow$ BCT precipitation in $\mathrm{L1}_{0}-\mathrm{TiAl}(\mathrm{Nb})$ alloy is listed in Table 3, which shows that the crystallographic features predicted by phase transforming invariant line theory are coincident well with those features measured by
experiments. So, it is obviously that the invariant line theory is effective in the prediction of the crystallography of $\mathrm{FCT} \rightarrow \mathrm{BCT}$ precipitation in $\mathrm{L1}_{0}-\mathrm{TiAl}(\mathrm{Nb})$ alloy.

According to the invariant line hypothesis, coherent needles lie on the cone of unextended lines as given by the stress-free transformation strain [29]. Any inextended line can be made an invariant line by the appropriate rotation. The coherent precipitate of $\gamma_{1}$ phase in this study was revealed to have a definite growth orientation of [001], which seems to derive that the distribution of coherent needles was restricted to [001] of the three [100] directions by FCT structure of the matrix. As a comparison, coherent Cr needles in Cu which are distributed on cones centered on the three $\langle 100\rangle$ directions [30].

### 3.2.5. Morphology of the $F C T \rightarrow B C T$ precipitation

One can calculate precise elastic energy based on elastic constants of two phases in the precipitating system according to Khachaturyan [31]. However, the elastic constants are currently absent for phase $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$. It is found that precipitate dimensions tend to be inverse to the directional mismatch at the initial stage of a precipitation sequence [32] and could be predicted based on optimum matching at the interface [33]. The morphology of $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ precipitating phase in $\mathrm{Ti}-\mathrm{Al}-\mathrm{Nb}$ alloy has been discussed roughly in the present author's work [11]. It is revealed that this precipitate takes a needle-like morphology, which coincides well with experimental observation.

There are three eigenvectors in Table 3 as a result of invariant line calculation. The rotation angle let the three real eigenvalues exist in this study. Luo and Weatherly implied that coherent inclusions tend to exhibit facets that are defined by unrotated directions (eigenvectors) of the transformation [25]. Thus we could deduce all the three eigenplanes bounding the precipitate phase shown as Fig. 5. The inclusion shape is predicted to be a parallelepiped with edges parallel to the eigenvectors and faces parallel to eigenplanes. The aspect ratio is inverse to the magnitude of the eigenstrains which is $\left|1-n_{3}\right| /\left|1-n_{1}\right| \cong 1 / 100$. That means the precipitate would take needle-like shape with a direction of [001]. The cross-sectional shape would be a parallelogram. The prediction coincides well with that of experimental observation.

As a comparison, the principle that precipitate dimensions tend to be inverse to the magnitude of the transformation strain [16] derived an elliptical cross-section. The proposal that interfaces are parallel to the planes of three independent dislocation loop arrays necessary to accommodate the transformation strain completely [34] would deduce that precipitate crosssections normal to the invariant line direction have three distinct facets. The characterization of morphology and crystallography of the precipitate reaction in $\gamma-\mathrm{TiAl} / \gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ couple strongly support the postulate that precipitates are

Table 2
Theoretical eigenvalues, eigenvectors, invariant line and interfaces for $\gamma \rightarrow \gamma_{1}$ transforming in TiAlNb alloy

| Phase transformation | Eigenvalues | Eigenvectors | Interfaces |
| :--- | :--- | :--- | :--- |
| $\gamma \rightarrow \gamma_{1}$ | $\lambda_{1}: 1.0000$ | $V_{1}:[1,1.0779,21.0672]$ | $F_{1}:(10.5335,10.5335,-1)$ |
|  | $\lambda_{2}: 1.00273$ | $V_{2}:[10.3035,10.3071,1]$ | $F_{2}:(26.5477,-26.6355,1)$ |
|  | $\lambda_{3}: 1.00277$ | $V_{3}:[1,-0.9999,0.0037]$ | $F_{3}:(1.0791,1,-21.4231)$ |

Table 3
The theoretical and experimental crystallography of $\gamma_{1}$ phase

| Phase <br> transformation | Crystallographic <br> characteristics | Theoretical values | Experimental values |
| :--- | :--- | :--- | :--- |
| $\gamma \rightarrow \gamma_{1}$ | O.R. | $(001)_{\gamma} / /(001)_{\gamma_{1}} ;[110]_{\gamma} 0.21^{\circ} \rightarrow[100]_{\gamma_{1}} ;$ | $(001)_{\gamma} / /(001)_{\gamma_{1}} ;[110]_{\gamma} / /[100]_{\gamma_{1}} ;$ |
|  |  | $[\overline{1} 10]_{\gamma} 0.21^{\circ} \rightarrow[100]_{\gamma_{1}}$ | $[\overline{1} 10]_{\gamma} / /[100]_{\gamma_{1}}$ |
|  | Habit plain | $F_{1}:(10.5335,10.5335,-1)$ | $(110)$ |
|  | Interface 2 | $F_{2}:(26.5477,-26.6355,1)$ | $(1-10)$ |
|  | Interface 3 | $F_{3}:(1.0791,1,-21.4231)$ | $(001)$ |
|  | Growing direction | $[1,1.0779,21.0672]$ | $[001]$ |

bounded by unrotated planes (eigenplanes) based on the experimental observation of three eigenplanes.

### 3.3. Morphology and crystallography of the $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ equilibrium phase

The literature [10] and [11] have shown the same crystallography and morphology of the $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ equilibrium phase in a $\mathrm{Ti}_{4} \mathrm{Al}_{3} \mathrm{Nb}_{9}$ alloy. However, the direction of the phase takes $\langle 111]$ compared with that of precipitated phase taking [001] revealed in this study. The departure of directions between them could also be explained well with invariant line theory. Dahmen [29] proposed that coherent needles lie on the cone of the unextended lines as given by the stress-free transformation strain while semicoherent needles lie along the intersection of the cone of unextended lines with a matrix slip plane.

Having established the lattice corresponding between matrix and precipitates, the coherent precipitated needles will grow further. Thus, the direction of the needle will be given by the solution of such simultaneous equations:
$\left\{\begin{array}{l}a X+b Y+c Z=0 \\ X^{2}+Y^{2}+Z^{2}=1 \\ \eta_{1}^{2} x^{2}+\eta_{2}^{2} y^{2}+\eta_{3}^{2} z^{2}=1\end{array}\right.$
where $\eta_{1}, \eta_{2}$ and $\eta_{3}$ are the main strain as showed in Eq. (1) and $a, b, c$ are the index of the slip plane for the matrix. For $\mathrm{L} 1_{0}$ structure, the slip planes are $\{101\}$. Let $a=1, b=0$, $c=-1$, we can get the desired direction of the needle-like equilibrium phase as $[0.579596,0.572833,0.579596]$ from


Fig. 5. A schematic showing the coherent precipitate is bounded by unrotated planes as well as the Bain orientation relationship between the matrix and the precipitate.

Eq. (6). The departure angle between [111] and [0.579596, $0.572833,0.579596$ ] is $0.02^{\circ}$. It is shown that the theoretical result coincides well with the experimental result.

## 4. Conclusion

(1) After elongation of ageing time, the microstructure of the alloy consists of matrix $\gamma-\mathrm{TiAl}$ and needle-like precipitate $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$. The precipitates take needle-like shape with growing direction of [001], the cross-sectional shape of parallelogram and the Bain orientation relationship to $(001)_{\mathrm{m}} / /(001)_{\mathrm{p}} ;[110]_{\mathrm{m}} / /[100]_{\mathrm{p}} ;[\overline{1} 10]_{\mathrm{m}} / /[100]_{\mathrm{p}}$.
(2) The crystallographic features and morphology of the precipitate predicted by three-dimensional phase transforming invariant line theory coincides well with those measured by experiments.
(3) The needle direction of [001] at early stage for coherent precipitate $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ and [111] for equilibrium phase $\gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ could be explained well based on the invariant line theory.
(4) The characterization of morphology and crystallography of the precipitate reaction in $\gamma-\mathrm{TiAl} / \gamma_{1}-\mathrm{Ti}_{4} \mathrm{Nb}_{3} \mathrm{Al}_{9}$ couple sustain the postulate that precipitates are bounded by unrotated planes (eigenplanes) with edges parallel to the eigenvectors when three real eigenvalues exist.

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