

# Near-Surface Scintillation ( $C_n^2$ ) Estimates from a Buoy Using Bulk Methods during EOPACE

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## 1. Summary

During the Electro-Optical Propagation Assessment in a Coastal Environment (EOPACE) experiment of August-September 1997, infrared transmission measurements were obtained along a 7 km path over San Diego Bay. Simultaneous meteorological measurements were obtained from a buoy located at the midpoint of the transmission path. In this study transmission-derived values of the refractive index structure parameter,  $C_n^2$ , are compared with bulk model-derived estimates obtained from the mean buoy data. The bulk  $C_n^2$  estimates agreed very well with the transmission measurements in unstable conditions. The bulk estimates were very poor in near-neutral conditions because of the great difficulty in accurately measuring the small air-sea temperature differences ( $\Delta T$ ) encountered, upon which  $C_n^2$  is highly dependent. The bulk  $C_n^2$  estimates agreed less well with the transmission measurements in stable conditions than unstable conditions. A theoretical sensitivity and error analysis shows that the bulk  $C_n^2$  estimates become extremely sensitive upon the measured value of  $\Delta T$  over a narrow Bowen ratio range and

under such conditions it is virtually impossible to accurately estimate  $C_n^2$  using bulk methods. The sensitivity analysis also indicates whether fluctuations in temperature, humidity or temperature-humidity correlation dominate the determination of  $C_n^2$  under various Bowen ratio ranges.

## 2. Introduction

Electro-optical (EO) imagery through the atmosphere near the ocean surface experiences rapid intensity fluctuations due to atmospheric turbulence, known as scintillation. Scintillation is closely related to the refractive index structure parameter,  $C_n^2$ . In an operational environment it would be useful to be able to evaluate and predict the effects of scintillation on EO imagery by estimating  $C_n^2$  from routinely measured air-sea parameters. Bulk models have been developed to estimate near-surface atmospheric turbulence properties from mean meteorological measurements. The relations between these atmospheric turbulence properties and  $C_n^2$  have also been established, thereby allowing  $C_n^2$  to be estimated from mean air-sea measurements. The purpose of

this study is to determine how accurately  $C_n^2$  can be estimated from routine meteorological measurements using bulk models under various conditions. This study is based on data obtained during the EOPACE experiment of August-September 1997. Bulk  $C_n^2$  estimates computed from mean measurements obtained on a buoy are compared with concurrent optical transmission-derived  $C_n^2$  measurements along an over-water propagation path to determine how closely the two methods agree under various air-sea conditions. In addition, a theoretical error analysis of the bulk  $C_n^2$  model is conducted.

### 3. Theory

The structure parameter for a quantity  $y$  is given by

$$C_y^2 = \frac{\overline{[y(x) - y(x+r)]^2}}{r^{2/3}}, \quad (1)$$

where  $y(x)$  and  $y(x+r)$  are the values of parameter  $y$  at two points separated by a distance  $r$  along the mean wind direction and the over-bar indicates an ensemble average. The refractive-index structure parameter,  $C_n^2$ , can be expressed according to the structure parameters for temperature,  $C_T^2$ , humidity,  $C_q^2$  and the temperature-humidity fluctuation correlation,  $C_{Tq}$ , as follows (Andreas [1]):

$$C_n^2 = A^2 C_T^2 + 2ABC_{Tq} + B^2 C_q^2, \quad (2)$$

where the coefficients  $A$  and  $B$  are known functions of the wavelength ( $\lambda$ ) and the mean atmospheric pressure ( $P$ ), temperature ( $T$ ), and specific humidity ( $q$ ). The first term on the right hand side of Eq. (2) represents temperature fluctuations and is always positive, the second term represents the correlation of temperature and humidity

fluctuations and can be positive or negative, while the third term represents humidity fluctuations and is always positive. For optical and infrared wavelengths the first term in Eq. (2) generally dominates, however, when the air-sea temperature difference is small the last two humidity-dependent terms can dominate.

### 4. The Bulk Surface-Layer Model

Monin-Obukhov similarity (MOS) theory is used to relate the structure parameters  $C_T^2$ ,  $C_q^2$  and  $C_{Tq}$  in Eq. (2) to the mean properties of the atmospheric surface layer. According to MOS theory, the fluxes of momentum, sensible heat and latent heat are assumed to be constant with height in the surface layer. In practice, the surface layer is regarded as the region near the surface where the fluxes vary by less than 10%, generally extending to a height of roughly 20 to 200 m. All dynamical properties in the surface layer are assumed to depend only upon the height above the surface,  $z$ , and upon certain scaling parameters, which are defined in terms of the assumed-constant fluxes, as follows:

$$u_* = (-\overline{w'u'})^{1/2}, \quad (3a)$$

$$T_* = -\frac{\overline{w'T'}}{u_*}, \quad (3b)$$

$$q_* = -\frac{\overline{w'q'}}{u_*}, \quad (3c)$$

where  $u_*$ ,  $T_*$  and  $q_*$  are the surface layer scaling parameters for wind speed, temperature and humidity, respectively, defined in terms of the kinematic fluxes of momentum ( $-\overline{w'u'}$ ), sensible heat ( $-\overline{w'T'}$ ), and latent heat ( $-\overline{w'q'}$ ), respectively. When a dynamical property is properly scaled by

these parameters, it can be expressed as a universal function of  $\xi$ , defined as:

$$\xi = \frac{z}{L} = \frac{zkg(T_* + 0.61Tq_*)}{Tu_*^2} \quad (4)$$

Here  $L$  is the Monin-Obukhov length scale,  $k$  is the von Karman constant ( $= 0.4$ ) and  $g$  is the acceleration due to gravity.  $\xi$  is often referred to simply as the 'stability', and is negative in unstable conditions, zero in neutral conditions, and positive in stable conditions.

According to MOS theory, the surface layer scaling parameters  $T_*$ ,  $q_*$  and  $u_*$  can be expressed in terms of the mean surface layer properties by the expression:

$$x_* = (\Delta x)k[\ln(z/z_{ox}) - \psi_x(\xi)]^{-1}, \quad (5)$$

where  $x$  represents wind speed ( $u$ ), temperature ( $T$ ) or specific humidity ( $q$ ) and the symbol  $\Delta$  denotes the mean air-sea difference. The  $\psi$  functions are the integrated dimensionless profile functions, defined by Paulson [2]. We have made the common assumption that  $\psi_T = \psi_q$ . The parameters  $z_{ou}$ ,  $z_{oT}$  and  $z_{oq}$  are known as the 'roughness lengths,' and were determined by the bulk surfaced-layer model formulated by Fairall et al. [3]. The reader is referred to this paper for further details on the bulk model employed in this study. We have assumed that the scalar roughness lengths are equal (i.e.  $z_{oT} = z_{oq}$ ).

When the structure parameters for temperature ( $C_T^2$ ), the temperature-humidity correlation ( $C_{Tq}$ ) and humidity ( $C_q^2$ ) are properly scaled according to MOS theory, they can be expressed as:

$$C_T^2 = T_*^2 z^{-2/3} g_T(\xi), \quad (6a)$$

$$C_{Tq} = T_* q_* z^{-2/3} g_{Tq}(\xi), \quad (6b)$$

$$C_q^2 = q_*^2 z^{-2/3} g_q(\xi), \quad (6c)$$

where  $g_T$ ,  $g_{Tq}$ , and  $g_q$  are dimensionless functions of  $\xi$  which must be determined empirically. Observations have not conclusively demonstrated that these functions are different from each other and MOS theory implies they should be similar [4]. Therefore, we have assumed that  $g_T = g_{Tq} = g_q \equiv g$ . Measurements of  $g$  for highly stable conditions ( $\xi > \sim 1$ ) are rare and exhibit much scatter. In this study we have used the function for  $g_T$  given by Andreas [1]:

$$g_T(\xi) = \begin{cases} 4.9(1 - 6.1\xi)^{-2/3}, & \xi \leq 0 \\ 4.9(1 + 2.2\xi^{2/3}), & \xi \geq 0 \end{cases} \quad (7)$$

Combining Eqs. (2), (5) and (6) results in:

$$C_n^2 = \frac{g(\xi)k^2 [A^2 \Delta T^2 + 2AB\Delta T\Delta q + B^2 \Delta q^2]}{z^{2/3} [\ln(z/z_{oT}) - \psi_T(\xi)]^2} \quad (8)$$

Combining Eqs. (4) and (5) results in:

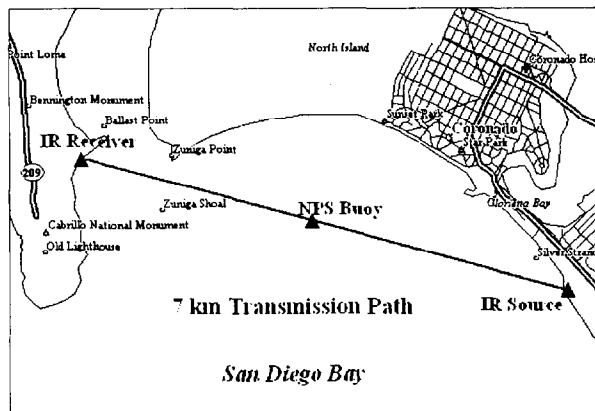
$$\xi = \frac{zg(\Delta T + 0.61T\Delta q)[\ln(z/z_{ou}) - \psi_u(\xi)]^2}{T\Delta u^2 [\ln(z/z_{oT}) - \psi_T(\xi)]} \quad (9)$$

$C_n^2$  can now be estimated from mean air-sea measurements by solving Eqs. (7), (8) and (9) by an iterative process.

## 5. Experimental Setup

The EOPACE experiment of August-September 1997 took place in San Diego Bay. Infrared (IR) transmission measurements were obtained by SPAWAR System Center, San Diego (SSC-SD). The transmission path was 7 km in length over

San Diego Bay, with the IR source (transmitter) located at the Naval Amphibious Base and the IR receiver located near the Bachelor Officers Quarters at the Naval Submarine Base (see Figure 1). Meteorological data were obtained concurrently with the transmission data from an instrumented buoy deployed by the Naval Postgraduate School (NPS) at the mid-point of the transmission path.



**Figure 1.** EOPACE experimental setup, August-September 1997.

## 6. Buoy Meteorological Measurements

The NPS buoy was deployed in San Diego Bay from 22 August to 8 September 1997. The following measurements were obtained on the NPS buoy: wind speed at a 4.9 m height above the surface, air temperature and humidity at 3.1 m, atmospheric pressure at 0.4 m, and sea temperature measured by a thermistor imbedded in the buoy hull 0.8 m below the surface. Only data obtained at night is included in this study, because it was discovered that solar radiation could penetrate the radiation shield and heat the temperature sensor, especially at low solar angles shortly after sunrise and before sunset. A fully enclosed, forced aspiration radiation shield could not be used on the buoy due to power constraints. The buoy measurements were averaged over 10 minute intervals and

bulk estimates of  $C_n^2$  were then computed from these mean values. The wavelength-dependent coefficients  $A$  and  $B$  in Eq. (8) were computed for a wavelength of  $3.8 \mu\text{m}$  using the formulas presented by Andreas [1]. Since  $C_n^2$  is height dependent (Eq. 8), the bulk  $C_n^2$  estimates were corrected for tidal sea level variations using tide data computed by the model 'Tides and Currents for Windows 95' by Nautical Software, Beaverton, OR.

## 7. Infrared Transmission Measurements

The SSC-SD transmission measurements were obtained from 23 August to 9 September 1997, using instruments and procedures similar to those described by Zeisse et al. [5]. The transmitter at the Amphibious Base was 6.2 m above mean sea level (MSL) and the receiver at the Submarine Base was 4.9 m above MSL. High-frequency mid-wave ( $3.5$  to  $4.1 \mu\text{m}$ ) IR transmission measurements were obtained hourly over a 41 second interval with a sampling frequency of 200 Hz. The lock-in time constant was 1 ms with a roll-off of 6 dB per octave (wait time 5 ms, equivalent noise bandwidth 250 Hz). The measured detector noise was less than 1 A/D level (0.1% of the free space signal) as compared to turbulent fluctuations between samples of about 50% of free space.  $C_n^2$  values were obtained by applying the normalized variance of the transmission data to the model formulated by Churnside et al. [6].

## 8. Bulk versus Transmission $C_n^2$ Comparison Results

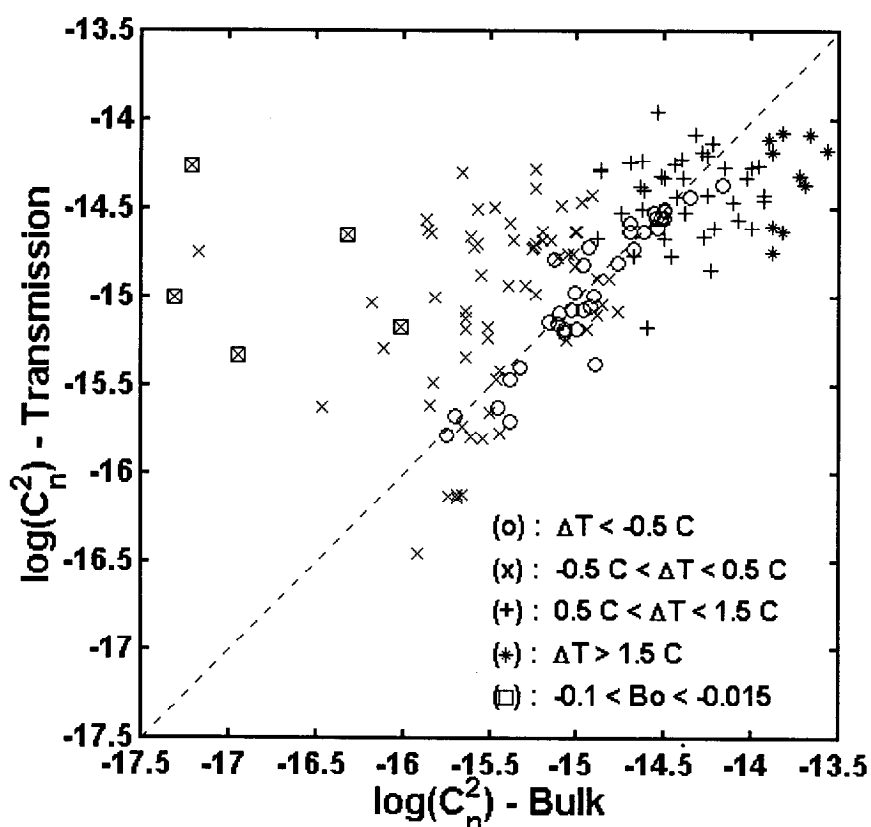
A scatter plot of the NPS bulk and SSC-SD transmission  $C_n^2$  values is presented in Figure 2. A summary of the comparison statistics is presented in Table 1. The data have been

separated into air-sea temperature difference ( $\Delta T$ ) intervals, as measured on the NPS buoy. The linear correlation coefficient between the two populations is presented in the second column. The ' % difference ' is the average value of  $(C_n^2(\text{trans}) - C_n^2(\text{bulk}))/C_n^2(\text{trans})$ . The 'rms % difference' is the value of  $[(C_n^2(\text{trans}) - C_n^2(\text{bulk}))^2]^{1/2}$ , where the brackets denote an average, divided by the mean value of  $C_n^2(\text{trans})$ .

**Table 1.** Bulk versus transmission  $C_n^2$  comparison statistics.

$\Delta T$ Range	Corr Coeff	% Diff	rms % Diff
$\Delta T < -0.5$ °C	0.93	-16	35
$-0.5$ °C $< \Delta T < 0.5$ °C	-0.05	33	358
$0.5$ °C $< \Delta T < 1.5$ °C	-0.02	-45	86
$1.5$ °C $< \Delta T$	0.28	-293	228

The agreement between the bulk and transmission  $C_n^2$  values is very good for unstable conditions ( $\Delta T < -0.5$  °C). For these conditions the percentage difference between the two methods is -16% and the correlation coefficient is 0.93. The agreement between the two methods is very poor for near-neutral conditions ( $-0.5$  °C  $< \Delta T < 0.5$  °C), exhibiting a very large degree of scatter (rms % difference of 358%), with the bulk  $C_n^2$  estimates being much lower than the transmission measurements in most cases. In weakly stable conditions ( $0.5$  °C  $< \Delta T < 1.5$  °C), the comparison between the two methods exhibits much more scatter than for unstable cases (rms % difference of 86% as compared to a rms % difference of 35% for unstable conditions). For strongly stable



**Figure 2.** Scatter plot of transmission  $\log(C_n^2)$  measurements versus bulk  $\log(C_n^2)$  estimates separated into air-sea temperature difference ( $\Delta T$ ) intervals:  $\Delta T < -0.5$  °C indicated by o's;  $-0.5$  °C  $< \Delta T < 0.5$  °C indicated by x's;  $0.5$  °C  $< \Delta T < 1.5$  °C indicated by +'s;  $\Delta T > 1.5$  °C indicated by \*'s. Data points within the Bowen ratio ( $Bo$ ) interval  $-0.1 < Bo < -0.015$  indicated by squares.

conditions ( $\Delta T > 1.5$  °C) the transmission  $C_n^2$  measurements are systematically much lower than the bulk estimates, by 293% on average. It is possible that the optical transmission data were 'saturated' for these very stable conditions, thereby causing the transmission values to be much lower than the bulk estimates.

### 9. Bulk $C_n^2$ Model Sensitivity and Error Analysis

A theoretical sensitivity and error analysis was conducted for the bulk  $C_n^2$  model used in this study, using methods similar to those of Andreas [1]. The sensitivity coefficient for  $C_n^2$  upon a parameter  $x$ ,  $S_x$ , can be defined as:

$$S_x = \frac{x}{C_n^2} \left[ \frac{\partial C_n^2}{\partial x} + \frac{\partial C_n^2}{\partial \xi} \frac{\partial \xi}{\partial x} \right]. \quad (10)$$

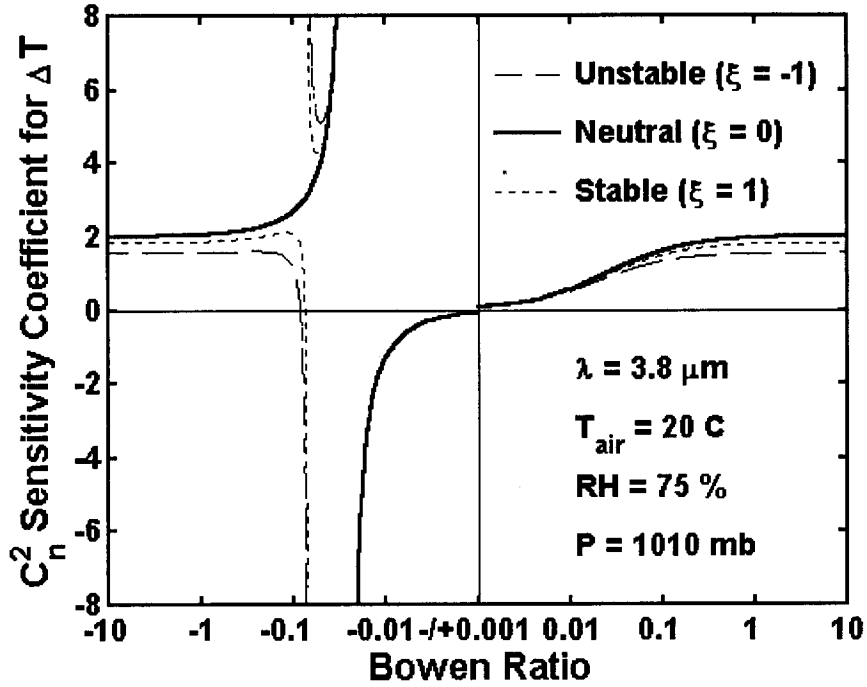
A large value of  $S_x$  indicates that the bulk  $C_n^2$  estimates are highly sensitive to the measured value of  $x$ , while a small value of  $S_x$  indicates that the bulk  $C_n^2$  estimates are virtually independent of  $x$ . The relative error in the bulk  $C_n^2$  estimates is given by multiplying  $S_x$  by the assumed relative error in the measurement of  $x$ :

$$\frac{\delta C_n^2}{C_n^2} = S_x \frac{\delta x}{x}. \quad (11)$$

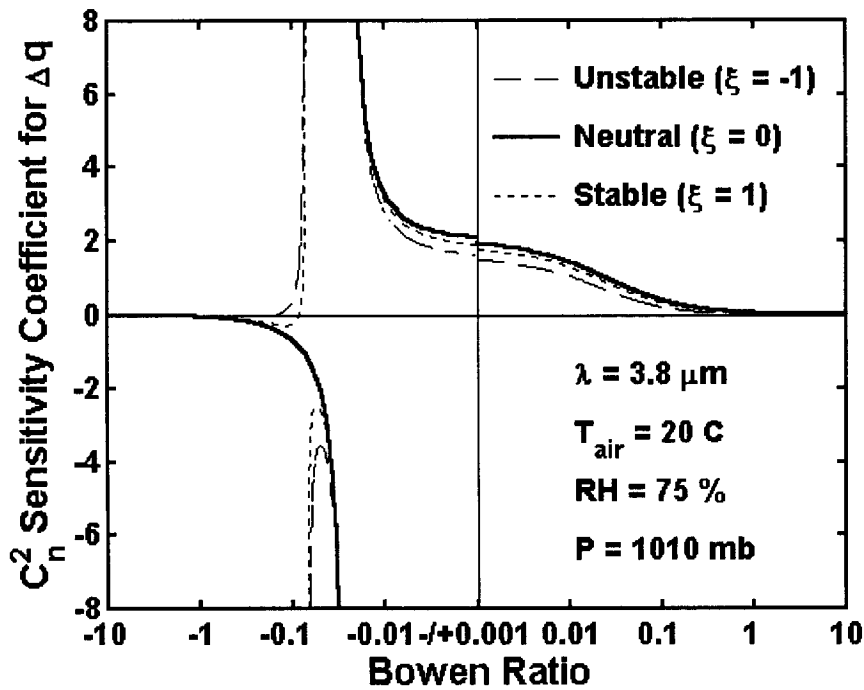
The  $C_n^2$  sensitivity coefficients for the air-sea temperature difference,  $S_{\Delta T}$ , and the air-sea humidity difference,  $S_{\Delta q}$ , are highly dependent upon the Bowen ratio. The Bowen ratio,  $Bo$ , is the ratio of the sensible heat flux over the latent heat flux, or  $Bo = c_p T^* / L_v q^*$ , where  $c_p$  is the specific heat of air at constant pressure and  $L_v$  is the latent heat of vaporization.  $S_{\Delta T}$  and  $S_{\Delta q}$  were computed for representative atmospheric conditions encountered during the experiment and for a

wavelength of 3.8  $\mu\text{m}$ , and are plotted versus  $Bo$  for different values of  $\xi$  in Figs. 3 and 4, respectively. Over a narrow Bowen ratio range (roughly  $-0.1 < Bo < -0.015$ ) the bulk  $C_n^2$  estimates become extremely sensitive upon the measured values of both  $\Delta T$  and  $\Delta q$ , making it virtually impossible to accurately estimate  $C_n^2$  by the bulk model in this  $Bo$  range. These small negative values of  $Bo$  generally only occur under near-neutral conditions (small  $T^*$  and, therefore,  $\Delta T$  values) and when the sensible and latent heat fluxes have opposite signs. In Figure 2 those data points with a Bowen ratio within the range  $-0.1 < Bo < -0.015$  are indicated by a square. It can be seen that all these data points occurred in near-neutral conditions (indicated by x's within squares) and that these bulk  $C_n^2$  estimates exhibited very large differences from the transmission  $C_n^2$  measurements, in agreement with the large predicted errors in the bulk  $C_n^2$  estimates within this  $Bo$  range.

From Figs. 3 and 4 it can be seen that, outside of the range  $-0.1 < Bo < -0.015$ , for a given value of  $Bo$  the absolute value of the sensitivity coefficients  $S_{\Delta T}$  and  $S_{\Delta q}$  are smallest for unstable conditions, larger for stable conditions and largest for neutral conditions. Therefore, for a given measurement accuracy the relative error in bulk  $C_n^2$  estimates will tend to be smallest for unstable conditions and largest for neutral conditions. This is in agreement with the bulk versus transmission  $C_n^2$  comparisons, with less scatter exhibited in unstable conditions (rms % difference of 35%) than in stable conditions (rms % difference of 86%) or near-neutral conditions. In near-neutral conditions the relative errors in  $C_n^2$  will tend to be very large on average because it is difficult to accurately measure the small  $|\Delta T|$  values encountered under such conditions, leading to large relative errors in  $\Delta T$ . This is reflected in the extremely large scatter in the



**Figure 3.**  $C_n^2$  sensitivity coefficient for the air-sea temperature difference,  $S_{\Delta T}$ , plotted versus the Bowen ratio. Unstable conditions ( $\xi = -1$ ) indicated by dashed line, neutral conditions ( $\xi = 0$ ) indicated by solid line and stable conditions ( $\xi = 1$ ) indicated by dotted line.



**Figure 4.**  $C_n^2$  sensitivity coefficient for the air-sea humidity difference,  $S_{\Delta q}$ , plotted versus the Bowen ratio. Unstable conditions ( $\xi = -1$ ) indicated by dashed line, neutral conditions ( $\xi = 0$ ) indicated by solid line and stable conditions ( $\xi = 1$ ) indicated by dotted line.

bulk versus transmission  $C_n^2$  comparison for near-neutral conditions (rms % difference of 358%).

The sensitivity coefficients  $S_{\Delta T}$  and  $S_{\Delta q}$  presented in Figs. 3 and 4 provide insight into the relative importance of temperature and humidity fluctuations in determining the resulting value of  $C_n^2$ . For  $Bo$  values less than  $-0.1$  and greater than  $0.1$ ,  $S_{\Delta T}$  is much greater than  $S_{\Delta q}$ , therefore temperature fluctuations, represented by the  $C_T^2$  term in Eq. (2), will dominate the resulting value of  $C_n^2$ . Within the range  $-0.01 < Bo < 0.01$ ,  $S_{\Delta q}$  is greater than  $S_{\Delta T}$ , therefore humidity fluctuations ( $C_q^2$ ) will dominate  $C_n^2$ . Within the range  $0.01 < Bo < 0.1$ ,  $S_{\Delta T}$  and  $S_{\Delta q}$  have similar magnitudes and are of the same sign, therefore the  $C_{Tq}$  term in Eq. (2) will dominate or be comparable in magnitude to  $C_T^2$  and  $C_q^2$ . Within the range  $-0.1 < Bo < -0.01$ ,  $S_{\Delta T}$  and  $S_{\Delta q}$  become very large and have opposite signs and it is impossible to accurately determine  $C_n^2$  or whether humidity or temperature fluctuations dominate.

## 10. Conclusions

This study has demonstrated that  $C_n^2$  can be accurately estimated in unstable conditions from routinely obtained meteorological measurements using bulk surface-layer models. The bulk  $C_n^2$  estimates were most accurate in unstable conditions because:

- 1) The bulk  $C_n^2$  estimates are the least sensitive to the measured values of  $\Delta T$  and  $\Delta q$  in unstable conditions.
- 2) The dimensionless structure function parameter,  $g(\xi)$ , upon which  $C_n^2$  is directly related, is better known in unstable conditions than in stable conditions.

Under neutral and stable stratification the use of bulk methods to estimate  $C_n^2$  was less successful. The poor accuracy observed in bulk  $C_n^2$  estimates in near-neutral conditions is due to three reasons:

- 1) The bulk  $C_n^2$  estimates are most sensitive to the measured values of  $\Delta T$  and  $\Delta q$  in neutral conditions.
- 2) The relative uncertainty in  $\Delta T$  measurements, which usually dominate the bulk  $C_n^2$  estimates, will tend to be largest in near-neutral conditions, when the values of  $|\Delta T|$  are smallest.
- 3) Over a narrow Bowen ratio range ( $-0.1 < Bo < -0.015$ ) which generally occurs only in near-neutral conditions, the bulk  $C_n^2$  values become extremely sensitive upon the measured values of  $\Delta T$  and  $\Delta q$ , making it impossible to accurately estimate  $C_n^2$ .

There are several probable reasons for the poor accuracy observed in bulk  $C_n^2$  estimates in stable conditions:

- 1) The bulk  $C_n^2$  estimates are more sensitive to the measured values of  $\Delta T$  and  $\Delta q$  in stable conditions than unstable conditions.
- 2) The dimensionless structure function parameter,  $g(\xi)$ , upon which  $C_n^2$  is directly related, is poorly known in stable conditions and the function in use may be greatly in error.
- 3) In very stable conditions turbulence is suppressed by the atmospheric stratification, which can allow the atmosphere to become de-coupled from the surface, thereby invalidating MOS theory and the bulk  $C_n^2$  model used in this study.



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