The Néel Theory of Ferrimagnetism

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The Néel theory of ferrimagnetism is a molecular-field theory of magnetic ordering in systems which contain nonequivalent substructures of magnetic ions. The two principal results of the theory are
(a) the curve of reciprocal susceptibility vs temperature above the Curie temperature is an hyperbola, and
(b) the curve of spontaneous magnetization vs temperature below the Curie temperature may have any one of a number of unusual forms. The exact form depends on the distribution of magnetic ions between the substructures and on the relative values of the molecular field coefficients.

The agreement between the predictions of the Néel theory and some experimental observations on ferrites is discussed briefly.

INTRODUCTION

Of the crystals which contain atoms or ions having permanent magnetic moments, a certain number exhibit the property of spontaneous magnetization. That is, interactions between the magnetic atoms result in their moments being largely aligned, so that the crystal develops a net macroscopic magnetic moment, even when no external field is applied. A spontaneously magnetized material is an example of a cooperative system, or one in which the constituents interact so strongly that they cannot, in any reasonable approximation, be assumed independent of each other. Other examples of cooperative phenomena are the ordering of alloys and the condensation of gases.

Substances which become spontaneously magnetized are generally divided into three classes: ferromagnetic, antiferromagnetic, and ferrimagnetic. The existence of the last two classes has been recognized only in recent years, and in both cases, the development of the basic concepts was principally due to L. Néel.\(^1\)\(^,\)\(^2\) The purpose of this paper is to review Néel’s theory of ferrimagnetism and to discuss briefly the agreement between theory and experiment. We begin by making a qualitative comparison of the magnetic properties of ferromagnetic, antiferromagnetic, and ferrimagnetic materials.

In a ferromagnetic material, the interactions between magnetic atoms favor a parallel alignment of the atomic magnetic moments. At absolute zero, the alignment is complete, all moments point in the same direction, and the spontaneous magnetization (i.e., magnetization in zero external field) has its maximum possible value. As the temperature is increased, the effects of thermal agitation, which favors random orientations of the magnetic moments, begin to be felt. Since the interactions favoring alignment remain constant, or nearly so, the spontaneous magnetization decreases with increasing temperature, gradually at first, and then more and more rapidly, until it becomes zero at a characteristic temperature known as the Curie temperature. Figure 1 shows the observed spontaneous magnetization-temperature curve for nickel; the corresponding curves for other ferromagnetic materials have similar shapes. Above the Curie temperature, where the effects of thermal agitation predominate, there is no spontaneous magnetization and the magnetic moments are oriented randomly. In this region, the material has many of the characteristics of a purely paramagnetic material.

In 1936, Néel\(^1\) suggested that another type of cooperative magnetization, which he called antiferromagnetism, might exist. In Néel’s original proposal, there were essentially two conditions necessary for the occurrence of antiferromagnetism. First, the interactions between

\(^1\) A condensed version of this paper was presented at the NOL Conference on Ferrimagnetism, October, 1954.
\(^2\) L. Néel, Ann. phys. 18, 5 (1932); Compt. rend. 203, 304 (1936).
\(^3\) L. Néel, Ann. phys. 3, 137 (1948).
nearest neighbor magnetic atoms should favor antiparallel, rather than parallel, alignment of the magnetic moments. Second, the crystal structure of the material should be such that the structure of magnetic atoms can be divided into two identical interpenetrating substructures; a magnetic atom on substructure 1 must have nearest neighbors on substructure 2 only, and vice versa. Then a possible type of magnetically ordered arrangement is to have substructures 1 and 2 magnetized antiparallel to each other. The spontaneous magnetization vs temperature curve for each substructure is like the curve shown in Fig. 1 for the ferromagnetic case; however, the net magnetic moment for the material is always zero (if no external field is applied). Above the Curie temperature or, as it is sometimes called, the Néel temperature, the material has paramagnetic characteristics.

Recent magnetic susceptibility and neutron diffraction experiments have demonstrated that Néel's hypothesis was essentially correct. In general, the arrangements are more complicated than the simple two-substructure model (some of the more complicated arrangements were also predicted by Néel in later papers) but the validity of the basic idea of identical substructures magnetized antiparallel to each other has been clearly demonstrated. For a detailed discussion of antiferromagnetism, the reader is referred to the several review articles in the literature, especially the recent one by Lidiard.\footnote{A. B. Lidiard, \textit{Reports on Progress in Physics} XVII (The Physical Society, London, 1984), p. 210.}

In 1943, Guillaud suggested that the magnetic properties of the compound Mn$_3$Sb, which were difficult to understand in the light of ordinary ferromagnetic theory, could be explained by the following assumptions: (a) Half of the Mn atoms were in the 3d$^7$ electronic state and half in the 3d$^{4s}$ electronic state. (b) Both magnetic substructures were spontaneously magnetized but in opposite directions, as in an antiferromagnet. However, Mn$_3$Sb should still have a net magnetization since the two kinds of Mn atoms have different magnetic moments and the perfect compensation that occurs in an antiferromagnet is lacking here. With this model, Guillaud was able to account for the observed saturation magnetization of Mn$_3$Sb and to qualitatively explain some of its other magnetic properties.

In 1948, Néel\footnote{C. Guillaud, thesis, Strasbourg, 1943.} proposed the existence of a third type of cooperative magnetic phenomena, of which Mn$_3$Sb was a special case, and worked out a simple but detailed theory of such phenomena. This third type, to which Néel gave the name ferrimagnetism, may be considered as a generalization of ferromagnetism and antiferromagnetism. In a ferrimagnetic material, the substructures into which the magnetic structure may be divided are not identical because the complete magnetic structure contains more than one kind of magnetic atom, or more than one kind of crystallographic site or both.\footnote{Different numbers of magnetic atoms on the substructures also contribute to the lack of symmetry; however, this possibility is usually intimately associated with one of the other two and is thus not considered separately.} The essential point is that certain interactions which would be equal if the substructures were identical are no longer equal. As we shall see, the introduction of this asymmetry is responsible for the special properties of ferrimagnetic materials.

In his 1948 paper, Néel also pointed out that the ferrite compounds, which had been developed as commercial magnetic materials by Snoek\footnote{J. L. Snoek, \textit{New Developments in Ferromagnetic Materials} (Elsevier Publishing Company, Inc., New York, 1947).} and his co-workers at Eindhoven, were ferrimagnetic. The great amount of research devoted

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Spontaneous magnetization of nickel as a function of temperature (after Weiss and Forrer).}
\end{figure}
to ferrimagnetism since that time has been due in part to the intrinsic properties of such materials and in part to the commercial importance of the ferrites.

**MOLECULAR FIELD APPROXIMATION**

The molecular field theory, which Néel uses in his treatment of ferrimagnetism, was originated by P. Weiss to explain the properties of ferromagnetic materials. Weiss suggested that the actual field acting on any atom in a ferromagnet should be written in the form

\[ H = H_0 + H_m, \]

(1)

where \( H_0 \) is the applied magnetic field and \( H_m \) is the internal or "molecular" field due to interactions with other atoms in the material. Thus the molecular field is the source of the cooperative behavior. Weiss then made the assumption that \( H_m \) is the same for all atoms and has the form

\[ H_m = \gamma M, \]

(2)

where \( M \) is the average magnetization of the material and \( \gamma \) is a constant, which we shall call the molecular field coefficient. This assumption, which has the virtue of simplicity, is obviously only a rough approximation for any actual crystal. However, when combined with the Langevin equation of state\(^9\) for a material containing noninteracting magnetic moments, the Weiss theory was qualitatively very successful in that it predicted: (a) a spontaneous magnetization below a certain temperature \( T_c \); (b) a temperature variation of the spontaneous magnetization which roughly resembled the curve of Fig. 1; and (c) a paramagnetic behavior above the Curie temperature. The agreement of the spontaneous magnetization-temperature curve with experiment was later much improved by a quantum-mechanical modification which replaced the Langevin function by a Brillouin function.

The great failure of the Weiss theory was quantitative. If the internal field resulted from ordinary magnetic dipole-dipole interactions, the maximum value that could be expected for the constant \( \gamma \) was \( 4\pi \) (Gaussian units); however, in order to obtain agreement with observed values of \( T_c \), values of \( \gamma \) of the order of \( 10^8 \) had to be assumed. It was not until the advent of the quantum theory that Heisenberg and others showed that the extremely high molecular fields were quantum mechanical in origin, and were a direct consequence of the restrictions placed on the wave functions by the Pauli principle. We do not propose to discuss this problem here, but rather to simply regard the molecular field coefficients as phenomenological constants.

In recent years, Néel\(^1\) and Anderson\(^11\) have modified the Weiss theory to take account of the actual structure of crystals. Their assumption may be written in the form

\[ H_i = H_0 + \sum_{k=1}^{n} \gamma_{ik} M_k. \]

(3)

Here the material is considered to be divided into substructures and \( H_i \) is the field acting on an atom on the \( i \)th substructure. The summation is over the different substructures, and \( \gamma_{ik} \) is the molecular field coefficient for the field exerted on an atom on the \( i \)th substructure by its neighbors on the \( k \)th substructure.

In the special case where the magnetic structure can be divided into identical substructures, this generalized molecular field theory predicts the existence of antiferromagnetism as well as ferromagnetism. One result is that the spontaneous magnetization of the \( i \)th substructure is given by

\[ M_i = N g \mu_B S B_i \left( \frac{3S}{S+1} \frac{T_c - M_i}{T_N g \mu_B S} \right), \]

(4a)

where \( N \) is the number of atoms per unit volume on one substructure, \( g \) is their Landé \( g \)-factor, and \( S \) their spin quantum number. \( B_i \) is the appropriate Brillouin function and \( \mu_B \) the Bohr magneton. Since \( N g \mu_B S \) is the saturation magnetization of a substructure, the equation may be rewritten in the simpler form

\[ \sigma_i = B_i \left( \frac{3S \sigma}{S+1} \right), \]

(4b)

where \( \sigma_i \) is the reduced magnetization of the

\(^9\) P. Weiss, J. phys. radium 4, 661 (1907).
\(^10\) See, for example, J. H. Van Vleck, Am. J. Phys. 18, 495 (1950).
ith substructure and \( T \) is the reduced temperature, \( T/T_c \). There is no simple explicit expression for \( \sigma \) as a function of \( \tau \), but the dependence is shown graphically in Fig. 2 for \( S = \frac{1}{2} \). The resemblance to the experimental curve in Fig. 1 is obvious.

The generalized theory also predicts that above the Curie temperature the reciprocal susceptibility is a linear function of temperature and has the form

\[
1/\chi = (T - \theta)/C.
\]

Here \( C \) is the Curie constant. For a ferromagnet, \( \theta = T_c \); for an antiferromagnet, \( \theta \) is usually negative and of the same order of magnitude as \( T_c \). Most experimental data on the high-temperature susceptibility of ferromagnets and antiferromagnets can be well represented by Eq. (5) with proper choice of the constants.

In the discussion of the Néel theory of ferrimagnetism, it will be instructive to compare the results obtained with the molecular field results for ferromagnetism and antiferromagnetism. The molecular field theory, of course, gives only approximate solutions, and thus one may question the wisdom of using its results for ferromagnetism and antiferromagnetism as a sort of “standard” with which to compare the ferrimagnetic case. However, we have seen that the molecular field theory does give results for ferromagnetism and antiferromagnetism which are qualitatively very good over the entire temperature range of interest, and, as we shall see, it gives correspondingly good results for ferrimagnetism. Thus, one may hope that the differences which the molecular field theory predicts between the ferrimagnetic and the other two cases will be at least qualitatively preserved in more exact treatments (which are valid only in a limited temperature range). This hope seems justified by the fact that most of the predicted differences have been observed experimentally.

As we have mentioned before, the ferrimagnetic case is distinguished from the other two by the fact that the material contains nonidentical substructures. The differences between the substructures may arise because there are different types of magnetic ions, different types of crystallographic sites for the ions or both. We shall consider only the special case of a ma-

![Fig. 2. Theoretical curve of reduced magnetization vs reduced temperature for \( S = \frac{1}{2} \).

\[ \gamma_{ab} = \gamma_{ba} \]

\[ H_{ab} = \gamma_{ab} M_a; \quad H_{ba} = \gamma_{ba} M_b \]

where \( H_{ab} \) is the molecular field acting on an \( A \) atom due to its \( B \) neighbors, with similar definitions for the other \( H \)'s. \( M_a \) and \( M_b \) are the magnetizations of the \( A \) and \( B \) substructures and the \( \gamma \)'s are molecular field coefficients. It can be shown from general arguments that \( \gamma_{ab} = \gamma_{ba} \) and that the coefficients are not altered if there are some nonmagnetic atoms on the \( A \) and \( B \) sites. However, \( \gamma_{ab} \neq \gamma_{ba} \) unless the \( A \) and \( B \) substructures are identical.

It turns out that a natural way of classifying the results obtained is according to whether \( \gamma_{ab} > 0 \) (favoring parallel alignment of \( M_a \) and

\[ \text{Such a distinction, though convenient, is somewhat artificial, since the observable magnetic properties of a given ion may depend upon the type of crystallographic site on which it is located.} \]
\( \mathbf{M}_a \) or \( \gamma_{ab} < 0 \) (favoring antiparallel alignment of \( \mathbf{M}_a \) and \( \mathbf{M}_b \)). For this reason, we shall find it convenient to consider \( \gamma_{ab} \) as a positive quantity and to write in a plus or minus sign explicitly to take care of the two possibilities mentioned above. The other two molecular field coefficients, \( \gamma_{aa} \) and \( \gamma_{bb} \), may be either positive or negative.

The total fields acting on \( A \) and \( B \) atoms are
\[
\mathbf{H}_a = \mathbf{H}_0 + \gamma_{aa} \mathbf{M}_a + \gamma_{ab} \mathbf{M}_b, \\
\mathbf{H}_b = \mathbf{H}_0 + \gamma_{ab} \mathbf{M}_a + \gamma_{bb} \mathbf{M}_b. 
\]

(7a,b)

It will also be convenient to define the quantities
\[
\alpha = \gamma_{aa}/\gamma_{ab}, \quad \beta = \gamma_{bb}/\gamma_{ab}, 
\]

(8a,b)

which give the ratios of the \( A-A \) and \( B-B \) interactions to the \( A-B \) interaction. Finally, we introduce the reduced magnetizations of each of the substructures:
\[
\sigma_a = \frac{\mathbf{M}_a}{\mu N \mu_B S} \lambda N \mu_B S, \\
\sigma_b = \frac{\mathbf{M}_b}{\mu N \mu_B S}. 
\]

(9a,b)

Then the molecular fields may be written in the alternate form
\[
\mathbf{H}_a = \mathbf{H}_0 + \mu N \mu_B S \gamma_{aa} \lambda [\alpha \sigma_a + \mu \sigma_b], \\
\mathbf{H}_b = \mathbf{H}_0 + \mu N \mu_B S \gamma_{ab} \lambda [\beta \sigma_a + \mu \sigma_b]. 
\]

(10a,b)

Then, according to molecular field theory, the thermodynamic equations of state for the material are:
\[
\sigma_a = B_2(\mu_B S \mathbf{H}_a/kT), \\
\sigma_b = B_2(\mu_B S \mathbf{H}_b/kT). 
\]

(11a,b)

These are a pair of simultaneous transcendental equations which must be solved for \( \sigma_a \) and \( \sigma_b \) as functions of \( T \) (or \( \tau \)). Since the methods of solution have no particular physical interest, we give only the results here. A few remarks about methods of obtaining numerical solutions will be found in the Appendix.

**A. Positive A-B Interaction**

We shall treat the case of positive \( A-B \) interaction first because it is simpler. (This is the reverse of the order in Néel's paper.) The appropriate expressions for \( \mathbf{H}_a \) and \( \mathbf{H}_b \) are then given by Eqs. (10) taken with the plus signs.

*Paramagnetic Susceptibility*

The reciprocal susceptibility at high temperatures is given by
\[
\frac{1}{T} = \frac{T - \theta}{T - \theta'} - \frac{1}{C}, 
\]

(12a)

where \( C \) is the Curie constant for \( N \) atoms. This expression is somewhat more complicated than the corresponding one [Eq. (5)] for ferromagnets and antiferromagnets, and the \( 1/\chi - T \) curve is a hyperbola instead of a straight line. Néel has shown that it may be written in the form
\[
\frac{1}{\chi} = \frac{T - \theta}{T - \theta'} - \frac{\xi}{C}, 
\]

(12b)

with
\[
\theta = \frac{C \gamma_a \lambda \mu [2 + \lambda \alpha / \mu + \mu \beta / \lambda]}{C \gamma_a \lambda \mu}, \\
\theta' = -\gamma_a \lambda \mu [2 - \alpha - \beta], \\
\xi = \gamma_a \lambda \nu [\lambda (1 - \alpha) - \mu (1 - \beta)]. 
\]

(13, 14, 15)

The \( 1/\chi - T \) hyperbola, with a geometrical interpretation of the parameters of Eq. (12b), is shown in Fig. 3.

If the \( A \) and \( B \) substructures are identical, then \( \lambda = \mu = \frac{1}{2} \) and \( \alpha = \beta \), in which case \( \xi = 0 \), \( \theta = T_c \) [see Eq. (16a) below], and the hyperbola reduces to two straight lines. Thus we find in this special case the usual results for ferromagnetism.

*Spontaneous Magnetization*

The numerator in Eq. (12a) may be factored into \( (T - T_c)(T - T'_c) \) where
\[
T_c = \frac{1}{2} C \gamma_{ab} \lambda \alpha + \mu \beta + [\lambda (\alpha - \beta)^2 + 4 \alpha \mu]^\frac{1}{2}, \\
T'_c = \frac{1}{2} C \gamma_{ab} \lambda \alpha + \mu \beta - [\lambda (\alpha - \beta)^2 + 4 \alpha \mu]^\frac{1}{2}. 
\]

(16a, 16b)

\[\text{FIG. 3. The Néel hyperbola.}\]

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\(^{12}\) Néel's (see reference 2) Eqs. (13) and (40) are printed incorrectly.
Thus the susceptibility becomes infinite at $T = T_c$ or $T = T'_c$; such behavior ordinarily indicates a Curie temperature. A detailed analysis shows that $T_c$ is the Curie temperature for an arrangement in which both substructures are spontaneously magnetized with $M_a$ and $M_b$ parallel; $T'_c$ is the Curie temperature for an arrangement in which $M_a$ and $M_b$ are anti-parallel. Since $T_c > T'_c$, we may expect that the parallel arrangement will occur first when the material is cooled from high temperatures, and that this state will be the stable state for all $T < T'_c$.\(^{14}\) (see, however, Yafet and Kittel.\(^{15}\) This result is consistent with our initial assumption that the $A - B$ interaction is positive.

The temperature variation of the spontaneous magnetizations below the Curie temperature is, of course, of particular interest. Since the molecular fields acting on the $A$ and $B$ substructures are different, we may expect that $M_a$ and $M_b$ will have different sorts of temperature variation. Such a situation is shown schematically in Fig. 4(a). In general, it is not possible to determine $M_a$ and $M_b$ independently by magnetization measurements. Rather what is observed is the total magnetization

$$M = M_a + M_b.$$  (17)

The corresponding reduced magnetization is defined by

$$\sigma = \lambda \sigma_a + \mu \sigma_b.$$  (18)

Since the most easily observed property is $M$ or $\sigma$, we shall follow Néel and classify the various types of spontaneous magnetization that can occur by the $\sigma - \tau$ curves.

As might be expected, the variation of $\sigma_a$, $\sigma_b$, and $\sigma$ with temperature is found to depend on $\lambda$, $\alpha$, and $\beta$. It is convenient to classify the different $\sigma - \tau$ curves by the regions in the $\alpha - \beta$ plane to which they correspond; here $\lambda$ is considered as an adjustable parameter. Figure 5 shows the partition of the $\alpha - \beta$ plane for the case $\lambda = \frac{1}{2}$. The boundary curves of the regions are

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\(^{14}\) J. S. Smart, Phys. Rev. 90, 55 (1953).

\(^{15}\) Y. Yafet and C. Kittel, Phys. Rev. 87, 290 (1952).
specially significant in determining the shape of the \( \sigma - \tau \) curves and we shall discuss each of them in turn.

We begin with the hyperbola

\[
\alpha \beta = 1. \tag{19}
\]

In the region \( G \) below and to the left of the negative branch of this hyperbola, the substructures do not become spontaneously magnetized at any real temperature, since \( T_c \) is negative in this region. The physical interpretation of this result is that since both \( \alpha \) and \( \beta \) are negative and at least one of the two is relatively large compared to unity, the interactions within the \( A \) and \( B \) substructures successfully oppose the tendency of the \( A-B \) interaction to spontaneous magnetization of each substructure. Néel characterizes this region as being always paramagnetic; however, this interpretation ignores the possibility of antiferromagnetic arrangements within one or both of the \( A \) and \( B \) substructures.

In the remainder of the \( \alpha - \beta \) plane, \( T_c > 0 \), and the two substructures will become spontaneously magnetized parallel to each other below \( T_c \). Obviously, the most favorable conditions for this arrangement exist when \( \alpha \) and \( \beta \) are positive or at least not too negative. If either \( \alpha \) or \( \beta \) has a large negative value (and \( (\alpha, \beta) \) does not lie in the region \( G \)), then the conditions are less favorable for reasons described in the preceding paragraph. In fact, according to Néel’s theory, the favorable and less favorable conditions can be quantitatively defined by their effects on the spontaneous magnetizations at absolute zero. Any point \( (\alpha, \beta) \) which lies above and to the right of the lines

\[
\alpha = -\mu / \lambda; \quad \beta = -\lambda / \mu \tag{20a,b}
\]

corresponds to a solution in which \( \sigma_a = \sigma_b = 1 \) at \( T = 0 \). That is, both substructures are saturated at absolute zero. On the other hand, if \( \alpha < -\mu / \lambda \), then \( \sigma_a < 1 \) at absolute zero and the \( A \) substructure is not saturated because of the negative \( A-A \) interactions. A corresponding situation exists for the \( B \) substructure if \( \beta < -\lambda / \mu \).

This subdivision leads to two different types of \( \sigma - \tau \) curves, as illustrated in Fig. 5. The type \( Q \) curves are generally similar to the usual Brillouin type of Fig. 2. In particular, the type \( Q \) curves have zero slope at \( T = 0 \) and a negative infinite slope at \( T = T_c \), two characteristic features of the Brillouin curve. If the point \( (\alpha, \beta) \) lies on the line

\[
\lambda (1 - \alpha) = \mu (1 - \beta) \tag{21}
\]

then one may show that \( |H_a| = |H_b| \) and \( \sigma_a = \sigma_b \) for all temperatures; then the \( \sigma_a - \tau, \sigma_b - \tau \), and \( \sigma - \tau \) curves all show the exact Brillouin behavior. If the point \( (\alpha, \beta) \) is not on the line but is still in the region \( Q \), the \( \sigma - \tau \) curve lies below the normal curve.

The \( R \)-type curves, in which one substructure is not saturated, have two principal differences from the \( Q \)-type curves. They are \( \sigma (0) \neq 1 \) and \( d\sigma /dT \neq 0 \) at \( T = 0 \) (resulting from corresponding conditions on either \( \sigma_a \) or \( \sigma_b \)). It should be pointed out immediately that these results cannot be completely correct, since the lack of saturation of one sublattice and the accompanying finite value of \( d\sigma /dT \) at \( 0^\circ K \) are forbidden by the third law of thermodynamics.14 Yafet and Kittel15 have suggested a modification of Néel’s theory which seems physically plausible and does not violate the third law of thermodynamics. Their suggestion was intended to apply specifically to the case of negative \( A-B \) interaction and so we defer discussion of it until later. However, we may note here that their arguments would have the same degree of validity for this situation.

Up to the present time, no actual materials are known which have the proper chemical, crystallographic, and magnetic properties to satisfy the conditions described above. As far as purely magnetic measurements are concerned, such materials would not be notably different.

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14 Consider a system whose equilibrium properties are specified by any two of \( M, H_b \), and \( T \). From the second law of thermodynamics, we can obtain

\[
\frac{dS}{T} = \frac{\partial M}{\partial T} + \frac{\partial M}{\partial H_b} \frac{dH_b}{T}.
\]

where \( S \) is the entropy. Then for any isothermal process

\[
\frac{dS}{\frac{\partial M}{\partial H_b}} \frac{dH_b}{T}.
\]

It is the sense of the third law that \( S \) is independent of \( H_b \) at \( T = 0 \). Then we must have

\[
\lim_{T \to 0} \frac{\partial M}{\partial H_b} = 0.
\]

The result may be generalized to include antiferromagnetic and ferrimagnetic systems.
from ordinary ferromagnetic substances, since the observable part of the $1/x-T$ hyperbola is nearly a straight line and the $\sigma-\tau$ curves have roughly normal behavior.

**B. Negative A-B Interaction**

We now turn to the case of negative $A-B$ interaction, which is both more interesting and more complicated than the case just considered. Also it has been realized experimentally.

**Paramagnetic Susceptibility**

The $1/x-T$ curve is also a hyperbola in this case and can be represented by a set of equations similar to Eqs. (12) to (15). The correct results can be obtained directly from Eqs. (12) to (15) by replacing $\gamma_{ab}, a,$ and $\beta$ by $-\gamma_{ab}, -a,$ and $-\beta$, respectively. For the special case, $\lambda=\mu=\frac{1}{2}$ and $a=b$, we find again $\zeta=0$ and the hyperbola reducing to a straight line (but $\theta\neq T_c$, this time).

These results may be shown to be equivalent to the molecular field results for antiferromagnetism.

**Spontaneous Magnetization**

We see that if the transformation $\gamma_{ab} \rightarrow -\gamma_{ab}, a \rightarrow -a, \beta \rightarrow -\beta$ is performed on Eqs. (16), the expressions for $T_c$ and $T'_c$ are merely interchanged. Thus the ordered state of lowest energy will be one in which both substructures are spontaneously magnetized with $M_a$ and $M_b$ antiparallel. This behavior is shown schematically in Fig. 4(b). Inspection of Eqs. (10) and (11) shows that for given values of $\gamma_{ab}, a,$ and $\beta$, $\sigma_a$ and $\sigma_b$ will be just the same functions of temperature as they were for the positive $A-B$ interaction case, except that now one is positive and the other is negative. Thus we may use the results already obtained as a guide in classifying magnetization temperature curves for negative $A-B$ interaction. For convenience we consider $\sigma_b$ to be positive and $\sigma_a$ to be negative. Equation (18) is still a valid expression for $\sigma$ but we should note that its maximum value is now $\lambda - \mu$ instead of unity.

The partition of the $\alpha-\beta$ plane for this case is shown in Fig. 6. The region in which $T_c<0$ and the regions in which one substructure is not saturated at $0^\circ K$ are the same as in Fig. 5.

![Fig. 6. Partition of $\alpha-\beta$ plane and types of $\sigma-\tau$ curves for negative $A-B$ interaction. Numbers in parentheses refer to equations in the text.](image)

We shall first consider the types of $\sigma-\tau$ curves which correspond to the upper right quadrant defined by Eqs. (20), i.e., the region in which $|\sigma_a|=|\sigma_b|=1$ at $T=0$. There are two important boundary lines in this region. The first, Eq. (21), again defines the points for which $|\sigma_a|$ and $\sigma_b$ have the exact Brillouin behavior. For points not on this line the curve of $\sigma$ [or more precisely, $\sigma/(\mu-\lambda)$] lies above or below the normal Brillouin curve according to whether the point $(\alpha, \beta)$ is above or below the line. Also, in the region above the line, $\sigma$ is a monotonic increasing function of $\tau$ in the neighborhood of absolute zero, while below this line, it is a monotonic decreasing function of $\tau$ for all temperatures; in either case, $d\sigma/dT \rightarrow 0$ as $T \rightarrow 0$. An increase of $\sigma$ with increasing $\tau$ may seem rather odd at first thought, but the reason is that $\mu \sigma_a$ decreases with $\tau$ less rapidly than does $\lambda |\sigma_a|$. Since $\sigma$ is the difference of these two quantities, it shows an increase. This behavior is demonstrated in Fig. 7, where calculated curves of $\sigma_a$, $\sigma_b$, and $\sigma$ are shown for the case $\lambda=\frac{1}{2}, \alpha=0, \beta=3$. It should be noted that $\sigma_a$ and $\sigma_b$ are always monotonic decreasing functions of $\tau$, regardless of the region of the $\alpha-\beta$ plane to which they correspond.

The condition

$$\lambda(1+\alpha)=\mu(1+\beta)$$

(22)
is the necessary condition that $d\sigma/dT \to 0$ as $T \to T_c$. (We have dropped the prime in order to conform with the usual notation.) For all points above this line, $\sigma$ has the direction of $\sigma_b$ just below the Curie temperature; for all points below the line, $\sigma$ has the direction of $\sigma_a$ just below the Curie temperature.

This division leads to two more types of $\sigma-\tau$ curves. In the region $Q$ between the two parallel lines the curves will have roughly the normal Brillouin form. Actually, the region $Q$ could be subdivided, since in its lower half, the $\sigma-\tau$ curves become concave upwards near the Curie temperature, although the condition $d\sigma/dT \to -\infty$ as $T \to T_c$ is maintained.

In the region $N$, we have $\sigma \parallel \sigma_a$ near the Curie temperature; on the other hand, we know that since $\lambda < \mu$, $\sigma \parallel \sigma_b$ at $T=0$. Thus somewhere between 0 and $T_c$, the net magnetization must go through zero, or, as Néel describes it, a compensation point occurs. Figure 8 shows this behavior schematically for the case $\lambda = \frac{3}{4}$, $\alpha = 1$, $\beta = -\frac{1}{4}$. In this region, and for that matter, for all points below Eq. (22), we have $d\sigma/dT \to 0$ as $T \to T_c$.

When we turn to the regions in which one substructure is not saturated at $0^\circ$K, still other types of $\sigma-\tau$ curves occur. In $M_1$, $\sigma$ is an increasing function of $\tau$ near absolute zero as in the type $P$; since the $A$ substructure is not saturated, $\sigma$ also has a finite slope at $T=0$.

Below the line $\beta = -\lambda/\mu$ lies the region $R$ in which $\sigma$ has a finite negative slope at $0^\circ$K, and the region $V$ which combines the same characteristic with a compensation point. Finally, below the line $\beta = -1$ (23) the value of $\sigma_b$ at $0^\circ$K becomes so small due to the negative $B-B$ interactions that $\sigma$ remains parallel to $\sigma_a$ at all temperatures. Then the compensation point disappears and the $\sigma-\tau$ curve has a type $M$ behavior.

We have previously mentioned that curves of the types $M$, $R$, and $V$ could not actually exist, since their behavior at absolute zero violates the third law of thermodynamics. Yafet and Kittel have suggested a way around this difficulty, which also gives more reasonable results for the paramagnetic region $G$. They begin by subdividing the $A$ and $B$ substructures in order to take better account of the interactions within substructures. This generalization leads them to predict certain types of magnetic ordering which do not occur in Néel's theory. In particular, they find that where large negative interactions exist within a substructure, say the $B$ substructure, the following arrangement may occur. There are two equivalent substructures $B_1$ and $B_2$ which are both spontaneously magnetized. Their magnetizations are not completely antiparallel but instead form an angle of less than $180^\circ$. This imparts a net magnetization to the $B$ substructure as a whole and this net magnetization is then set antiparallel to $M_a$. Figure 9b shows this "triangular" arrangement schematically. The triangular arrangements give the same order of magnitude of $\sigma$ as Néel's unsaturated substructure model, but in addition they have a lower free energy and exhibit the correct thermodynamic properties near absolute zero.
Yafet and Kittel's results, which are applicable for both positive and negative $A-B$ interactions, may be summarized briefly with Fig. 5 as a reference. (The definitions of $\alpha$ and $\beta$ must be modified but they still remain measures of $A-A$ and $B-B$ interactions.) In the quadrant $Q$ of Fig. 5, Néel's results remain unchanged for both cases. In the region $G$, doubly antiferromagnetic arrangements occur, with each substructure becoming antiferromagnetically ordered at its characteristic Curie temperature. [Fig. 9(c)]. In the regions $R$ of Fig. 5, the triangular arrangements replace Néel's unsaturated substructures as the type of ordering which occurs at $0^\circ$K. The behavior in this region is complicated by the fact that transitions to ferromagnetic and doubly antiferromagnetic arrangements may occur at higher temperatures. Lotgering\textsuperscript{17} has made a detailed analysis of the types of magnetization curves which occur in Yafet and Kittel's theory.

Thus Yafet and Kittel's proposal is very attractive because of the simple and logical way in which it removes some serious objections to Néel's results. Some experimental evidence relevant to Néel's theory and the Yafet and Kittel modification are discussed in the next section.

**EXPERIMENTAL RESULTS ON FERRITES**

Néel\textsuperscript{3} pointed out that the magnetic properties of the ferrites should provide an excellent experimental test of his theory. The ferrites are compounds with the chemical formula $\text{MFe}_2\text{O}_4$, where $\text{M}$ is a divalent metallic atom. They have the spinel crystal structure in which the oxygen ions form a cubic close-packed arrangement with the metallic ions in the interstices between oxygens. There are two different types of interstices, one in which the metallic ion is surrounded by four oxygen ions at the vertices of a tetrahedron and one in which the metallic ion is surrounded by six oxygen ions at the vertices of an octahedron. Since the occupied tetrahedral and octahedral sites occur in the ratio 1:2, we shall henceforth refer to them as $A$ and $B$ sites, respectively.

Thus the ferrites contain two nonidentical magnetic substructures because of the two differ-

\textsuperscript{17} F. Lotgering (unpublished work).
accurately by hyperbolas as predicted by Eqs. (12). As an example, the data obtained by McGuire\textsuperscript{18} on the compound CoCr\textsubscript{2}O\textsubscript{4}, which has the spinel structure, are shown in Fig. 10.

From Eq. (12b), we see that at sufficiently high temperatures, the slope of the $1/\chi - T$ curve should be $1/C$. It has been reported several times that the experimental data on various compounds do not support this prediction. Néel himself seems to have adopted this point of view and has sought to explain the supposed discrepancy by assuming a temperature variation of the molecular field coefficients.\textsuperscript{18} However, we feel that the question is still open, since there exist two complicating factors whose detailed effects are not well enough understood to make a quantitative comparison of theory and experiment meaningful. One factor which may account for some of the discrepancy is that a given magnetic ion may have different magnetic properties on an $A$ site than it does on a $B$ site.\textsuperscript{20} This is because the electric fields from the oxygen ions are different for the $A$ and $B$ sites and thus the orbital contributions to the magnetic moment of the ion may be different for the two sites. Also the distribution of magnetic ions between $A$ and $B$ sites may be a function of temperature.\textsuperscript{20,21} This last factor by itself produces a nonhyperbolic $1/\chi - T$ curve; combined with the first factor mentioned above, it leads to a Curie "constant" which varies with temperature.

One very interesting point about the ferrites is of course whether the $A-B$ interaction is positive or negative. In principle this point can be most easily determined from the saturation magnetizations, since the maximum value of $\sigma$ is unity in one case and $(\mu - \lambda)$ in the other. We see that if the $A-B$ interaction is negative, the maximum value of $\sigma$ depends on the distribution of magnetic ions. At low temperatures, most ferrites are practically completely inverted, i.e., they have Fe\textsuperscript{3+} ions on $A$ sites, and Fe\textsuperscript{2+} and Mn\textsuperscript{2+} ions on $B$ sites. The experimental magnetization data usually gives the magnetic moment, $\mu$, of a "molecule" Fe\textsubscript{2}O\textsubscript{4}. In this case we should have for an inverted ferrite,

$$\mu_{\pm} = (\mu_{Fe})_{B} + (\mu_{M})_{B} \pm (\mu_{Fe})_{A},$$  \hspace{1cm} (24)

depending on whether the $A-B$ interaction is positive or negative. The Fe\textsuperscript{3+} ion has an $S$ state as ground state so that its orbital angular momentum is zero, and the crystalline electric fields affect its magnetic properties very little. Thus, we may assume $\mu_{Fe}$ is 5 Bohr magnetons for both $A$ and $B$ sites. The values of $(\mu_{M})_{B}$ are not known very accurately but they may be expected not to deviate too far from the "spin only" values, $2S_{M}\mu_{B}$, where $S_{M}$ is the spin quantum number of ion $M$. As we shall see, this assumption is sufficiently accurate to enable us to distinguish between the two possibilities in Eq. (24).

Using the assumptions above and writing all moments in Bohr magnetons, Eq. (24) becomes

$$\mu_{+} = 10 + 2S_{M}$$ \hspace{1cm} (25a)

$$\mu_{-} = 2S_{M}. \hspace{1cm} (25b)$$

Néel pointed out that for magnetite, Fe\textsubscript{3}O\textsubscript{4}, $S_{M} = 2$, $\mu_{+} = 14$, and $\mu_{-} = 4$. The experimental value\textsuperscript{22} is 4.08, indicating a negative $A-B$ interaction. Measurements of the saturation magnetization of ferrites with $M = $ Mn, Fe, Co, Ni, and Cu were made by Gorter\textsuperscript{23} and Guillaud and co-workers.\textsuperscript{24} Their results are shown in Table I.

\textsuperscript{18} McGuire, Howard, and Smart, Ceramic Age 60, No. 12, 222 (1952).

\textsuperscript{19} L. Néel, J. phys. radium 12, 238 (1951).

\textsuperscript{20} L. S. Smart, Phys. Rev. 94, 847 (1954).

\textsuperscript{21} L. Néel, Compt. rend. 230, 190 (1950).

\textsuperscript{22} P. Weiss and R. Forrer, Ann. phys. 12, 279 (1929).


\textsuperscript{24} C. Guillaud and M. Roux, Compt. rend. 229, 1133 (1949); C. Guillaud and H. Crevaux, Compt. rend. 230, 1256 (1950); C. Guillaud, J. phys. radium 12, 239 (1950).
The existence of an antiparallel alignment of $M_a$ and $M_b$ must be inferred in each case. Two arguments have been offered to explain the small differences between $\mu_a$ and $\mu_b$; (a) The ferrites are not completely inverted, and (b) the g-values of the $M^{++}$ ions are different from 2, i.e., there are orbital contributions to the magnetic moment. These suggestions seem physically reasonable; moreover, they are supported qualitatively by experiments on the change of $\mu$ with quenching and measurements of g values by susceptibility and magnetic resonance methods. The discrepancies between the $\mu$-values obtained by the two authors are attributed to differences in chemical composition and ionic distribution caused by the methods of preparation and to differences in the limiting temperatures and fields attainable by the measuring equipment.

Thus the experiments of Gorter and Guillaud clearly supported Néel's theory in that they indicated the existence of a negative $A-B$ interaction, or more precisely, of an antiparallel alignment of $M_a$ and $M_b$. More evidence on these points was provided by the neutron diffraction experiments of Shull, Wollan, and Strauser, which agreed with Néel's model much better than with other models, including others which gave the same saturation magnetization. Later, by studying the polarization of neutrons reflected from various planes in a magnetized Fe$_2$O$_3$ single crystal, Shull showed unambiguously that $M_a$ and $M_b$ were antiparallel. Now antiparallelism of $M_a$ and $M_b$ certainly strongly suggests a negative $A-B$ interaction, and Néel, Gorter, and Guillaud had previously interpreted it as meaning such. However, it is perhaps not conclusive, since there are some other rather farfetched possibilities which would give the same arrangement. Shull was able to rule out such possibilities by showing that if the net magnetization $M$ were rotated by rotating the external magnetic field, $M_a$ and $M_b$ also rotated with the field, remaining antiparallel and parallel, respectively, to it. Shull's experiment provides conclusive evidence that there is an $A-B$ interaction and that it is negative.

Since the $A-B$ interaction is negative, Néel's theory predicts that $\sigma - \tau$ curves for the ferrites will be of types $Q, P$, or $N$ if $\sigma(0) = (\mu - \lambda)^{27}$ When Néel's paper was written, the existing experimental data showed only $Q$, or "normal," $\sigma - \tau$ curves. However, since that time, discovery of the two anomalous types has been announced, thus confirming these details of Néel's theory. Magnetization curves in which $\sigma$ increases with $\tau$ at low temperatures (type $P$) have been found by McGuire for NiCrO$_3$ and by Maxwell and Pickart and Gorter for solid solutions of NiFe$_2$O$_4$ and NiAl$_2$O$_4$ in the approximate ratio 2:1. Magnetization curves in which $\sigma$ changes sign at a compensation temperature (type $N$) have been found by Gorter for certain Li-Fe-Cr spinels. Typical experimental curves for $P$ and $N$ types are shown in Figs. 11 and 12.

Gorter and Schulkes have devised a simple but elegant experiment which demonstrates the existence of compensation points. A rod of type $N$ material is magnetized along its axis and

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**TABLE I. Saturation moments of single ferrites.**

<table>
<thead>
<tr>
<th>M</th>
<th>$S_M$</th>
<th>$\mu_a$</th>
<th>$\mu_b$</th>
<th>$\mu_{(obs)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn</td>
<td>5/2</td>
<td>15</td>
<td>5</td>
<td>5.0</td>
</tr>
<tr>
<td>Fe</td>
<td>2</td>
<td>14</td>
<td>4</td>
<td>4.2</td>
</tr>
<tr>
<td>Co</td>
<td>3/2</td>
<td>13</td>
<td>3</td>
<td>3.3</td>
</tr>
<tr>
<td>Ni</td>
<td>1</td>
<td>12</td>
<td>2</td>
<td>2.3</td>
</tr>
<tr>
<td>Cu</td>
<td>1/2</td>
<td>11</td>
<td>1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

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$^{27}$ This expression must, of course, be generalized for materials containing more than one kind of magnetic ion.  
$^{28}$ T. R. McGuire (unpublished work).  
suspended so that it is free to rotate in a horizontal plane. A horizontal magnetic field is applied to align the rod in a given direction; this applied field should be kept considerably smaller than the coercive force of the material so that the magnetic state of the rod will not be altered appreciably. Then, when the rod is heated or cooled through the compensation temperature, it rotates through 180°, showing that the direction of the net magnetization of each crystallite in the material has reversed with respect to the crystal axes. A convenient material to use for this experiment is Li₁₁₁₁Fe₇₄₄₄ Cr₆₄₄₄O₄ as it has a compensation point about 12° above room temperature.

In a number of cases, \( \sigma - \tau \) curves have been found which look like the forbidden types \( M \) and \( R \). Presumably, the slopes of these curves would have to change at some sufficiently low temperature in order to conform with the third law of thermodynamics. To investigate this point, McGuire and Howard\(^\text{22}\) measured the magnetization of Ferrocube III, a commercial Mn-Zn ferrite, which has such a pronounced type \( R \) behavior that the \( \sigma - \tau \) relation is nearly linear. Figure 13 shows the temperature variation of the magnetization of Ferrocube III between the Curie temperature and 4°K. The curve shows no signs of deviating from type \( R \) behavior in this temperature range; any change in slope must come below 4°K.

Another question of considerable interest is whether any evidence for the doubly antiferromagnetic and triangular arrangements has been observed in ferrites or related compounds. For double antiferromagnetism, the answer is clear; no evidence for it has been found. This result is not surprising; purely geometric arguments suggest that the parameter \( a \) for a spinel structure will generally be so small that the point \( (a, 0) \) will not fall in region \( G \).

The existence of triangular arrangements at low temperatures is suggested if the inequality \( |\sigma(0)| < |\mu - \lambda| \) obtains. Gorter\(^\text{29}\) has studied several systems of mixed oxides, e.g., MnFe\( _{2-a} \)Cr\( _a \)O\( _4 \), 0 ≤ \( a \) ≤ 2, in which he finds that the inequality above occurs over at least part of the composition range. Gorter also shows that if triangular arrangements do exist in these compounds, the angles are on the \( B \) sites, a result which seems reasonable from geometric considerations. Thus, there is some indirect evidence for triangular arrangements; however, preliminary neutron diffraction results obtained by Corliss and Hastings\(^\text{38}\) on MnCr\( _2 \)O\( _4 \) are not in agreement with Gorter's model, although other triangular configurations may be possible. Also, Anderson\(^\text{34}\) has pointed out that the problem of

\(^{22}\) T. R. McGuire and L. N. Howard (unpublished work).

Fig. 12. Magnetization vs temperature for Li₁₁₁₁Fe₇₄₄₄Cr₆₄₄₄O₄ (after Gorter). Type \( N \).

Fig. 13. Magnetization vs temperature for Ferrocube III (after McGuire and Howard). Type \( R \).

\(^{29}\) L. Corliss and J. Hastings, NOL Conference on Ferrimagnetism, October, 1954.

\(^{34}\) P. W. Anderson, NOL Conference on Ferrimagnetism, October 1954.
order on the $B$ sites is complicated by the topological properties of the $B$ sublattice. These properties are analogous to those of the plane hexagonal lattice, in which the system remains disordered at all temperatures (Ising model). Thus, at the present, the question of the existence of triangular arrangements in spinel structures is still open. However, if there are no triangular arrangements, then the occurrence of the inequality $|\sigma(0)| < |\mu - \lambda|$ must be explained in some other way.

In addition to discussing the agreement between theory and experiment, we should see what information of theoretical interest might be deduced from the experimental data. One might hope that values of $\alpha$, $\beta$, and $\gamma_{ab}$ could be obtained for different compounds and enough correlations observed to make it possible to predict these quantities with reasonable accuracy for a given material having the spinel structure. Such results would not only be of value in solid-state theory but would also be of practical interest in designing commercial ferrite materials. In principle, $\alpha$, $\beta$, and $\gamma_{ab}$ can be determined from either the susceptibility data or the spontaneous magnetization data. In practice, these determinations are complicated by such factors as two or more kinds of magnetic ions, different magnetic properties on $A$ and $B$ sites for a given ion, change of ionic distribution with temperature, variations of coefficients with lattice parameters, and a number of others. Because of these complications, it seems fair to say that all of the empirically determined values of $\alpha$, $\beta$, and $\gamma_{ab}$ must be regarded as arbitrary, and that any correlations observed are probably meaningless. A limited amount of valid and useful information could probably be obtained by making careful studies of some very simple systems such as magnesium ferrite, in which the number of complicating factors is small.

In general, the Néel theory has been strikingly successful in predicting the qualitative features of the magnetic properties of ferrites, especially considering the extreme simplicity of the theory and the detailed nature of its predictions. Its principal quantitative success has been in giving the correct saturation magnetizations. Future improvements in the theory of ferrimagnetism will probably come from replacing the molecular field theory by more exact methods, rather than from refining Néel’s model.

**WHAT IS FERRIMAGNETISM?**

It may be well to conclude by asking just what we mean by ferrimagnetism. Néel originally introduced the term to apply specifically to the ferrites. However, he noted that the two most distinguishing magnetic properties of the ferrites, the hyperbolic $1/\chi - T$ curves and the abnormal $\sigma - T$ curves, were consequences of the existence of nonidentical magnetic substructures with negative interactions between substructures. Thus, all such materials should be called ferrimagnetic, and it is apparently in this sense that the word is used in most of the current literature. But we also have the question of what to call materials with nonidentical substructures and positive $A-B$ interactions; such substances also have the hyperbolic $1/\chi - T$ curves and abnormal $\sigma - T$ curves, but the departures from "normality" are ordinarily much less than in the case of negative $A-B$ interaction. There are three reasonable possibilities for naming such materials; we may (a) call them ferromagnetic (as is usually done), (b) call them ferrimagnetic, i.e., use nonidentical substructures as the sole criterion for ferrimagnetism, or (c) invent a new name. Concerning this last suggestion, it would seem logical to refer to materials having positive $A-B$ interaction as ferrimagnetic and to materials having negative $A-B$ interaction as antiferromagnetic; however, the word ferrimagnetism is too finely embedded in the literature on ferrites to make this suggestion practical now. We have no particular recommendation on nomenclature; it is important, though, that the conceptual differences between the various possibilities be clearly understood.

**ACKNOWLEDGMENTS**

I am indebted to the Brookhaven National Laboratory for its hospitality during the summer of 1954, when this paper was written. I am also

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*G. H. Wannier, Phys. Rev. 79, 357 (1950).*
indebted to Dr. R. K. Wangsness for discussions and suggestions.

**APPENDIX**

In Appendix II of his paper, Néel gives a graphical method of finding \( \sigma_a \) and \( \sigma_b \). Here we wish to outline a simple numerical method which gives good accuracy (three decimal places) with relatively little labor for the case \( S = \frac{1}{2}, \ B_4(x) = \tanh x \). We write Eqs. (11) in the form

\[
\begin{align*}
\sigma_a &= \tanh \left[ \frac{\alpha \lambda \sigma_a \pm \mu \sigma_b}{\tau F(\lambda, \alpha, \beta)} \right] \\
\sigma_b &= \tanh \left[ \frac{\pm \lambda \sigma_a + \beta \mu \sigma_b}{\tau F(\lambda, \alpha, \beta)} \right]
\end{align*}
\] (26a,b)

where

\[
F(\lambda, \alpha, \beta) = T_c / C_\gamma s_b
\]

\[
= \frac{1}{2}[\lambda \alpha + \mu \beta + [\lambda(\alpha - \mu \beta)^2 + 4 \lambda \mu]^{1/2}].
\]

Near the Curie temperature, roughly in the region \( 0.95 \leq \tau \leq 1 \), solutions may be obtained directly from an approximation developed by Néel.

\[
\sigma_a^2 = \rho \sigma_b^2 = \phi(S) \rho G(\lambda, \rho)(1 - \tau), \quad (1 - \tau) \ll 1,
\]

(27)

where

\[
\begin{align*}
\phi(S) &= \frac{10}{3} \frac{(S+1)^2}{S^2 + 5^2} \phi(\frac{1}{2}) = 3 \\
G(\lambda, \rho) &= [\lambda \rho + \mu / \rho] / [\lambda \rho^2 + \mu / \rho^3]
\end{align*}
\]

(28) (29)

and \( \rho \) is a root of

\[
\lambda \rho^2 + (\lambda \alpha - \mu \beta) \rho - \mu = 0.
\]

(30)

Rho is the positive or negative root depending on whether the plus or minus sign is used. If the solutions obtained in this way do not give consistent results when substituted into Eqs. (26), the agreement can be improved by iterative procedures.

For \( \tau < 0.95 \), values of \( \sigma_a \) and \( \sigma_b \) for a given \( \tau \) can be estimated from the portions of the \( \sigma_a - \tau \) and \( \sigma_b - \tau \) curves already computed. These estimated values are then inserted in Eqs. (26) and the agreement improved by iteration as before.

The equations

\[
\begin{align*}
\lambda \sigma_a &= \frac{\tau F(\lambda, \alpha, \beta)}{(\alpha \beta - 1)} \left[ \pm \text{tanh}^{-1} \sigma_a \mp \text{tanh}^{-1} \sigma_b \right] \\
\mu \sigma_b &= \frac{\tau F(\lambda, \alpha, \beta)}{(\alpha \beta - 1)} \left[ \mp \text{tanh}^{-1} \sigma_a \pm \text{tanh}^{-1} \sigma_b \right]
\end{align*}
\] (31a,b)

are also useful for numerical computation if tables of the archtanh are available.

These procedures also can be applied, of course, for \( S > \frac{1}{2} \), but since the tables for \( B_6(x) \) are less complete than those for \( \tanh x \), the results are less accurate in general.

**Horizons' Postdoctoral Fellowship**

The establishment of a fellowship at the famed Cavendish Laboratory at the University of Cambridge, England, was announced by Horizons Incorporated, the unique industrial and governmental research organization of Cleveland. The postdoctorate grant is for work in the field of physics and chemistry of the solid state and will be done under the guidance of Professor N. F. Mott, director of the Cavendish Laboratory. Fees, tuition, living expenses, and other emoluments usually associated with such study grants are covered in the fellowship.

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