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Propagation of coherently combined flattened laser beam array in turbulent atmosphere

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ABSTRACT

The analytical expression for the propagation of a flattened laser beam array in a turbulent atmosphere is derived based on extended Huygens–Fresnel principle. The influence of beam order and turbulent atmosphere on beam quality is studied. It is revealed that the beam quality of a coherently combined laser beam array with higher order is better than its lower order counterpart when propagating in free-space, weak and medium turbulence (i.e. $C_n^2 < 10^{-13}\,m^{-2/3}$). The beam quality of higher order beam arrays degrades faster as the intensity of turbulence gets stronger. In the case of propagating in strong turbulence, the beam order has no influence on coherently combined beam quality.

1. Introduction

The coherent combining of laser beams, which has been under intense research recently [1–4], has the potential to solve the power limitation in single laser caused by medium inhomogeneity, mode control, heat management and offers the ability to excite large gain volumes as well as maintaining good beam quality. The propagation of coherently combined laser beams in free space, through paraxial optical system or in turbulent atmosphere, has been modeled and discussed extensively to show the advantages and wide application potential [5–9]. Most previous research focused on the properties of single-mode Gaussian beam arrays, which unavoidably leads to a loss of beam quality compared to fully filled, uniform (i.e. flat-top) near field [1,10]. Coherent combining four ~100 W fiber beamlets distributed in a 2×2 laser array was reported in [3], yet only 63% of the whole laser power is contained within the far-field central main-lobe. The encircled far-field central main-lobe energy can be substantially increased by shaping the near-field energy distribution of radiation from each element in the laser array to be flattened beams [4,11]. Although high-power flattened beams generation is not so easy for today's laser technology, however, the coherent combining of a flattened laser beam array has the potential and may finally lead to applications such as long-range energy delivering. To the best of our knowledge, the propagation of a flattened laser beam array in a turbulent atmosphere has not yet been discussed. In the present paper, based on the extended Huygens–Fresnel principle, we derive analytical formulae for average intensity distributions in receiving plane. The propagation properties of coherently combined flattened laser beam array and the influence of beam order, intensity of turbulence will be discussed in detail.

2. Theories

2.1. Propagation formula for flattened laser beam array

We assume that the coherently combined flattened laser beam array is located at a source plane ($z=0$). The laser beams propagate along the z axis in the Cartesian coordinate system. The optical field of the beam array can be expressed as

$$E(x,y,z=0)=\sum_{i=1}^{M}E_i(x,y,z=0) \tag{1}$$

where $E_i(x,y,z=0)$ denotes the ith flattened beamlet in the laser array, which can be written as [12–14]

$$E_i(x,y,z=0)=\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n}\times\exp\left\{-\frac{n[((x-a_i)^2+(y-b_i)^2)]}{w_0^2}\right\} \tag{2}$$

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Here w_0 denotes the waist radius of the flattened laser beam and (a_i,b_i) is the center of the ith beamlet located at the source plane, $\binom{N}{n}$ denotes a binomial coefficient and N is the order of the flattened beams. When $N=1$, Eq. (2) reduces to a single-mode Gaussian beam.

By using the extended Huygens–Fresnel principle, the average intensity of the coherently combined flattened laser beam array at the z-plane after propagating in the turbulent atmosphere can be expressed as [8,9,12,14]

$$\begin{aligned}\langle I_N(p,q,z)\rangle &= \frac{k^2}{(2\pi z)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle E(x,y,0)E^*(\xi,\eta,0)\rangle \\ &\times \exp\left\{\frac{\mathrm{i}k}{2z}[(p-x)^2+(q-y)^2-(p-\xi)^2-(q-\eta)^2]\right\} \\ &\times \langle \exp[\psi(x,y,p,q)+\psi^*(x,y,p,q)]\rangle\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\xi\,\mathrm{d}\eta \end{aligned} \tag{3}$$

where $k=2\pi/\lambda$ is the wave vector modulus (lambda is the laser wavelength). (p,q) denotes the transverse coordinate at the recover plane, $\psi(x,y,p,q)$ denotes the random part of the complex phase of a spherical wave that propagates from the source point to the receiver point, the angle brackets indicates the ensemble average over the medium statistics covering the log-amplitude and phase fluctuations due to the turbulent atmosphere. In this paper, Kolmogorov spectrum and a quadratic approximation of the 5/3 power law for Rytov's phase structure function is employed. The last term in the integrand of Eq. (3) can be written as [8,9,12,14]

$$\langle \exp[\psi(x,y,p,q)+\psi^*(\xi,\eta,p,q)]\rangle = \exp\left\{-\frac{1}{\rho_0^2}[(x-\xi)^2+(y-\eta)^2]\right\} \tag{4}$$

Here $\rho_0=(0.545C_n{}^2k^2{}_z)^{-3/5}$ is the coherence length of a spherical wave propagating in turbulent atmosphere with $C_n{}^2$ being the structure constant.

We can express the average intensity of the coherently combined flattened laser beam array at the z-plane in the following form:

$$\langle I_N(p,q,z)\rangle = \sum_{n=1}^{M}\sum_{m=1}^{M}\sum_{l=1}^{N}\sum_{s=1}^{N}\Gamma_{mnls}(p,q,z) \tag{5}$$

where

$$\begin{aligned}\Gamma_{nmls}(p,q,z) &= \frac{k^2}{(2\pi z)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(-1)^{l+s-2}}{N^2}\times\binom{N}{l}\binom{N}{s} \\ &\times \exp\left\{-\frac{n[(x-a_n)^2+(y-b_n)^2]}{w_0^2} - \frac{m[(x-a_m)^2+(y-b_m)^2]}{w_0^2}\right\} \\ &\times \exp\left\{\frac{\mathrm{i}k}{2z}[(p-x)^2+(q-y)^2-(p-\xi)^2-(q-\eta)^2]\right\} \\ &\times \exp\left\{-\frac{1}{\rho_0^2}[(x-\xi)^2+(y-\eta)^2]\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\xi\,\mathrm{d}\eta \end{aligned} \tag{6}$$

By using the integral formula Eq. (3.323.2) of Ref. [15] and performing the related integral we can obtain

$$\begin{aligned}\Gamma_{nmls}(p,q,z) &= \frac{\rho_0^4k^2}{4z^2}\frac{(-1)^{l+s-2}}{N^2}\times\binom{N}{l}\binom{N}{s}\frac{1}{\beta_1\beta_2\rho_0^4-1} \\ &\times \exp\left\{-\frac{k^2\rho_0^2(\beta_1\rho_0^2+\beta_2\rho_0^2-2)[(p+a_n)^2+(q+b_n)^2]-4z^2\beta_1\rho_0^4\left[\frac{m^2(a_{nm}^2+b_{nm}^2)}{w_0^4}-\frac{im(a_{nm}+b_{nm})}{w_0^2z}\left(\frac{1}{\beta_1\rho_0^2}+1\right)\right]}{4z^2(\beta_1\beta_2\rho_0^4-1)}\right\}\end{aligned} \tag{7}$$

where $\beta_1=(l/w_0^2)+(1/\rho_0^2)-(\mathrm{i}k/2z)$, $\beta_2=(s/w_0^2)+(1/\rho_0^2)-(\mathrm{i}k/2z)$, $\mathrm{a}_{nm}=\mathrm{a}_n-\mathrm{a}_m$, $\mathrm{b}_{nm}=\mathrm{b}_n-\mathrm{b}_m$. Eq. (7) is the main analytical result of the present paper, which presents a convenient and effective way for studying the propagating properties of coherently combined flattened laser beam array in the turbulent atmosphere.

If we set $N=M=1$, the average intensity of single Gaussian beams propagating in turbulent atmosphere can be obtained

$$\begin{aligned} I(p,q,z) &= \frac{\rho_0^4k^2}{4z^2}\frac{1}{\beta_1\beta_2\rho_0^4-1} \times \exp\left\{-\frac{k^2\rho_0^2(\beta_1\rho_0^2+\beta_2\rho_0^2-2)(p^2+q^2)}{4z^2(\beta_1\beta_2\rho_0^4-1)}\right\} \\ &= \frac{k^2w_0^4\rho_0^2}{k^2w_0^4\rho_0^2+4z^2(2w_0^2+\rho_0^2)} \times \exp\left[-\frac{2k^2w_0^2\rho_0^2(p^2+q^2)}{k^2w_0^4\rho_0^2+4z^2(2w_0^2+\rho_0^2)}\right]\end{aligned} \tag{8}$$

It agrees with the existing results of Gaussian beams in turbulent atmosphere [16].

2.2. Beam quality factors

In order to give a more quantitative study, beam quality factors should be used to study the influence of beam order, intensity of turbulence on coherently combined beam qualities. Beam quality factors such as M^2 factor and Strehl ratio have been extensively used to characterize laser beam quality of single laser beam [17,18]. However, it has been calculated and proven that those beam quality factors are not always convenient to characterize the beam quality of coherently combined beams [19,20]. We consider

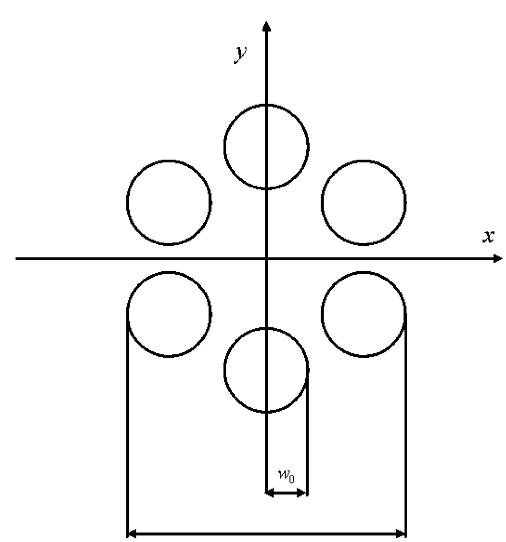

Fig. 1. Schematic diagram of the radial laser beam array.

that the beam quality of the coherently combined beam can be better characterized by the beam propagation factor (BPF) [21], which can be calculated by using the following equation: $\mathrm{BPF} = P/P_{\mathrm{DL}}$. Where P is the laser output power in a specified far-field bucket, and P_{DL} is the total output power radiating from the effective near-field exit aperture of the laser beam array. According to Ref. [21], the far-field bucket is defined as $A = (\pi/4)(\theta_z)^2$, which is the diffraction-limited bucket, $\theta = 2.44\lambda/D$, D is the effective exit aperture of the combined laser beam. BPF focus on the energy encircled in a certain bucket (basically the main-lobe), in general, BPF is smaller than 1. The closer BPF is to 1, the better beam quality is.

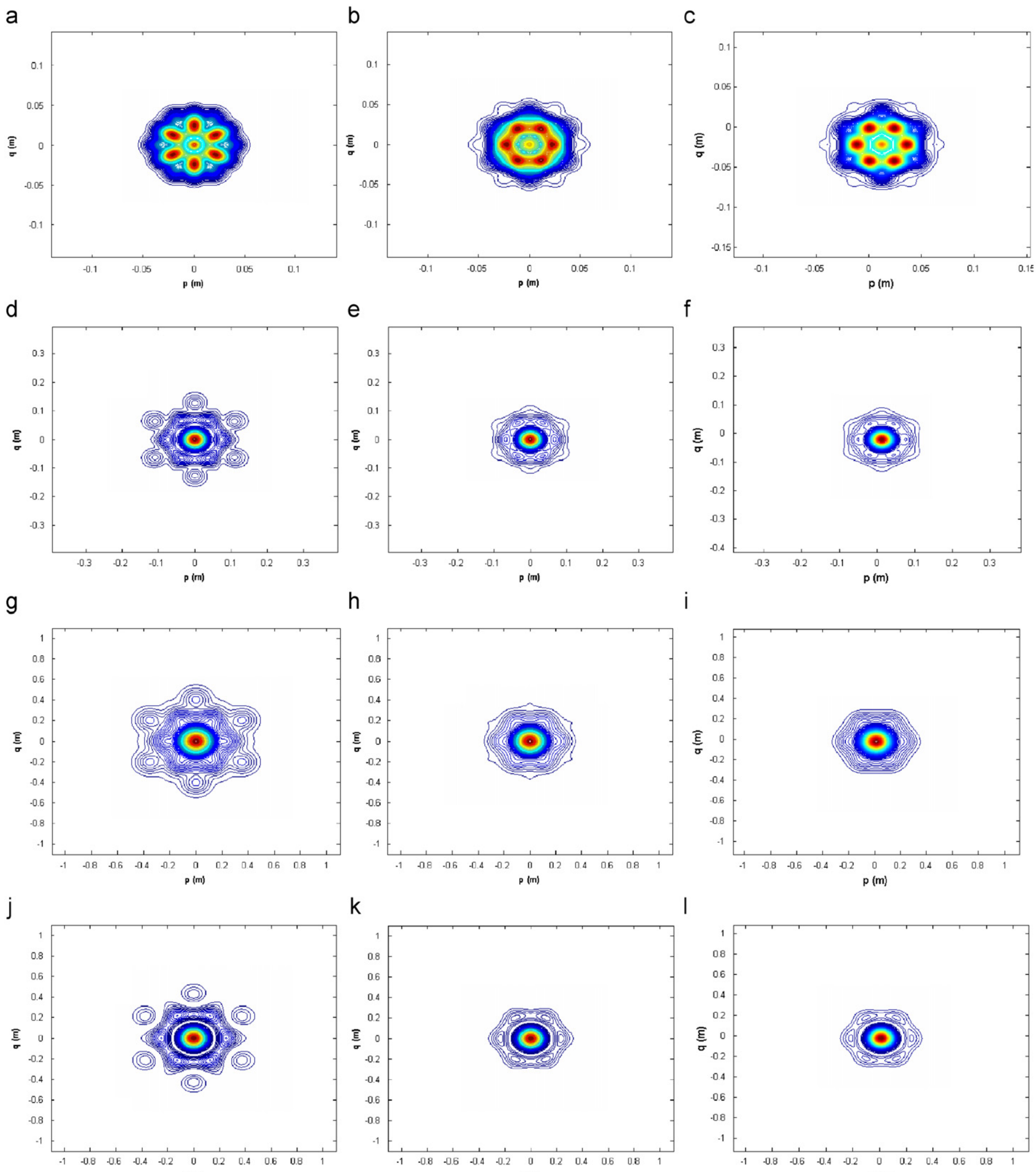

Fig. 2. Average intensity distribution of coherently combined flattened beam array with different orders at several propagation distances.

3. Numerical calculation and analysis

In this section, we study the propagation properties of coherently combined flattened laser beam array in turbulent atmosphere and the influence of beam order and intensity of turbulence on coherently combined beam quality.

We consider a radial laser beam array located symmetrically at the source plane with six equal beamlets, as shown in Fig. 1. The parameters used in Fig. 1 are chosen as $\lambda = 1.5\,\mu m$, $w_0 = 1\,cm$, $D = 7\,cm$.

Fig. 2 shows the average intensity distribution of coherently combined flattened beam array with different orders at several propagation distances. The propagation distances are selected to be 0.5, 3, and 10 km, the structure constant is chosen as $C^2{}_n = 10^{-15}\,m^{-2/3}$. The figures at the left side are for flattened laser beam array with order $N = 1$ (i.e. single-mode Gaussian beam), figures at the middle and right side are for flattened laser beam array with order $N = 6$ and 10, respectively. The average intensity distribution for a coherently combined laser beam array propagating in free space is also plotted as a reference case. From Fig. 2 one sees that the intensity distribution of coherently combined single-mode Gaussian beam array gradually evolves from multiple-petal-like shape into the pattern that contains one main-lobe in the center with multiple side-lobes. For the case of a coherently combined single-mode Gaussian beam array propagating in free space, there is no energy distributed in the large-area gaps between the main-lobe and the side-lobes when the propagation distance is of $z = 10\,km$. While for a flattened laser beam array with higher order ($N = 6$ and 10), the intensity distribution gradually evolves from multiple-petal-like shape into the pattern that contains only one main-lobe with several side-rings. The coherently combined flattened laser beam with higher beam order is less expanded when propagating in turbulent atmosphere with $C_n{}^2 = 10^{-15}\,m^{-2/3}$, as a result more energy will be encircled in the main-lobe. The BPF value for Fig. 2 (g)–(i) are calculated to be 0.60, 0.78 and 0.81, respectively.

(a) $N = 1$, $z = 0.5\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(b) $N = 6$, $z = 0.5\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(c) $N = 10$, $z = 0.5\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(d) $N = 1$, $z = 3\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(e) $N = 6$, $z = 3\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(f) $N = 10$, $z = 3\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(g) $N = 1$, $z = 10\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(h) $N = 6$, $z = 10\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(i) $N = 10$, $z = 10\,km$, $C^2{}_n = 10^{-15}\,m^{-2/3}$,
(j) $N = 1$, $z = 10\,km$, $C^2{}_n = 0$,
(k) $N = 6$, $z = 10\,km$, $C^2{}_n = 0$,
(l) $N = 10$, $z = 10\,km$, $C^2{}_n = 0$.

The dependence of BPF on the intensity of a turbulent atmosphere is shown in Fig. 3. On the whole, whatever the order of the flattened laser beam array, BPF value will decrease with increasing intensity of the turbulent atmosphere. One can see from Fig. 3 that the turbulent atmosphere has almost no influence on the beam quality when propagating in weak turbulence (i.e. $C_n{}^2 < 10^{-15}\,m^{-2/3}$). Beam quality degrades rapidly when propagating in medium intensity turbulence (i.e. $1 \times 10^{-15}\,m^{-2/3} < C_n{}^2 < 5 \times 10^{-14}\,m^{-2/3}$), the beam quality of a flattened laser beam array with higher beam order is on the whole better than the ones with lower order. For the strongest levels of turbulence intensity (i.e. $C_n{}^2 < 1 \times 10^{-13}\,m^{-2/3}$), the beam quality is nearly the same for flattened laser beam array with any beam order. Fig. 4 shows the dependence of BPF on beam orders. Since more optical energy is encircled in the central main-lobe for a higher order beam array in different atmospheric environments, it can be concluded generally that the beam quality of a flattened laser beam array with higher order is better than their counterparts with lower order. The degradation of beam quality of a flattened laser beam array with higher order is faster than the lower order array when the intensity of turbulence gets stronger. The beam quality of the flattened laser beam array with higher order is no better than its lower order counterpart when $C_n{}^2 = 1 \times 10^{-13}\,m^{-2/3}$. This can be explained by the fact that for a laser beam array propagating in strong turbulence, beam radius expansion is mainly caused by turbulence, which is independent of the source coherence and the beam radius of the intensity distribution across the source plane (see Eq. (7) in [22]). Fig. 3 together with Fig. 4 presents a basic understanding of the propagating properties in turbulent atmosphere with different beam orders.

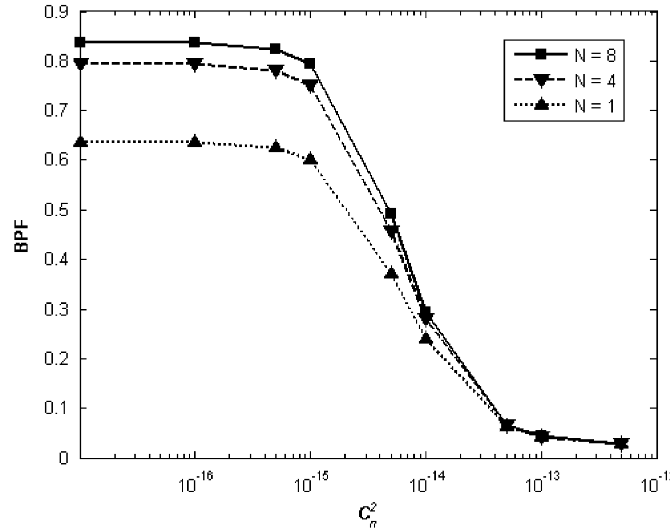

Fig. 3. The dependence of BPF on the intensity of turbulent atmosphere.

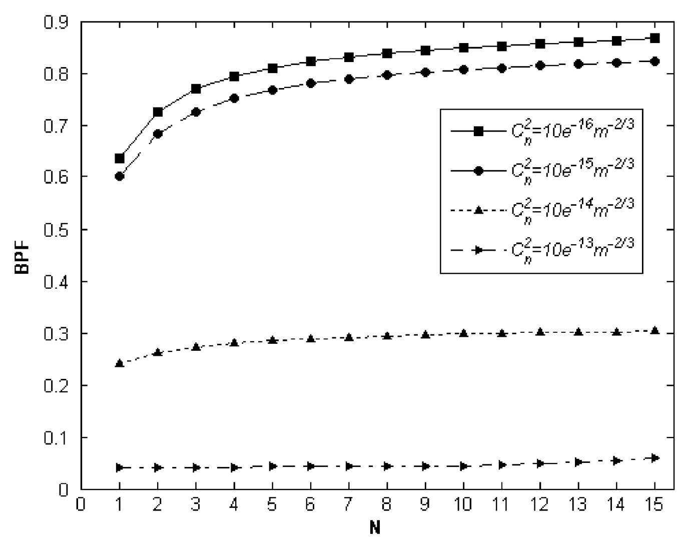

Fig. 4. The dependence of BPF on beam orders.

4. Conclusions

In the present paper, the analytical expression for propagation of a flattened laser beam array in turbulent atmosphere is derived.

The influence of beam order and turbulent atmosphere on beam quality is studied. A radially distributed laser array is taken as an example and numerical calculation of the average intensity distribution at several given propagating distance is performed. From the investigation we can see that the beam quality of coherently combined laser beam array with higher order is better than its lower order counterpart when propagating in free-space, weak and medium turbulence. However, the beam quality of coherently combined laser beam array with higher order degrades faster as the intensity gets stronger. In the case of propagating in strong turbulence, there is almost no difference in beam quality for coherently combined flattened beam array with different orders.

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References

[1] Fan TY. Laser beam combining for high-power, high-radiance sources. IEEE J Sel Top Quantum Electron 2005;11:567–77.
[2] Anderegg J, Brosnan S, Cheung E, Epp P, Hammons D, Komine H, et al. Coherently coupled high power fiber arrays. In: Proceedings of SPIE, vol. 6102, 2006. p. 61020U-1–5.
[3] Augst SJ, Ranka JK, Fan TY, Sanchez A. Beam combining of ytterbium fiber amplifiers. J Opt Soc Am B 2007;24:1707–15.
[4] Goodno GD, Asman CP, Anderegg J, Brosnan S, Cheung EC, Hammonsm D, et al. Brightness-scaling potential of actively phase-locked solid-state laser arrays. IEEE J Sel Top Quantum Electron 2007;13(3):460–72.
[5] Lü B, Ma H. Coherent and incoherent combinations of off-axis Gaussian beams with rectangular symmetry. Opt Commun 1999;171:185–94.
[6] Lü B, Ma H. Beam propagation properties of radial laser arrays. J Opt Soc. Am A 2000;17:2005–9.
[7] Li Y, Qian L, Lu D, Fan D, Wen S. Coherent and incoherent combining of fiber array with hexagonal ring distribution. Opt Laser Technol 2007;39:957–63.
[8] Chu X, Liu Z, Wu Y. Propagation of a general multi-Gaussian beam in turbulent atmosphere in a slant path. J Opt Soc Am A 2008;25:74–9.
[9] Cai Y, Chen Y, Eyyuboglu HT, Baykal Y. Propagation of laser array beams in a turbulent atmosphere. App Phy B 2007;88:467–75.
[10] Cheung EC, Ho JG, Goodno GD, Rice RR, Rothenberg J, Thielen P, et al. Diffractive-optics-based beam combination of a phase-locked fiber laser array. Opt Lett 2008;33:354–6.
[11] Brosnan SJ, Wichham MG, Komine H. Method and apparatus for optimizing the target intensity distribution transmitted from a fiber coupled array. US patent 72283702; 2007.
[12] Chu X, Ni Y, Zhou G. Propagation analysis of flattened circular Gaussian beams with a circular aperture in turbulent atmosphere. Opt Commun 2007;274: 274–80.
[13] Wu G, Lou Q, Zhou J, Dong J, Wei Y, Su Z. Propagation of flat-topped beams 2008;40:494–8.
[14] Alavinejad M, Ghafary B, Kashani FD. Analysis of the propagation of flat-topped beam with various beam orders through turbulent atmosphere. Opt Lasers Eng 2008;46:1–5.
[15] Gradysteyn IS, Ryzhik IM. Tables of integrals, series and products. New York: Academic Press; 1980.
[16] Eyyuboğlu HT, Baykal Y. Reciprocity of cos-Gaussian and cosh-Gaussian laser beams in turbulent atmosphere. Opt Express 2004;12:4659–74.
[17] Ji X, Lu B. Turbulence-induced quality degradation of partially coherent beams. Opt Commun 2005;251:231–6.
[18] Shealy DL, Hoffnagle JA. Laser beam shaping profiles and propagation. Appl Opt 2006;45:5118–31.
[19] Siegman AE. How to (maybe) measure laser beam quality. Presented at OSA annual meeting; 1998.
[20] Zhou P, Liu Z, Xu X, Chen Z. Numerical analysis of effects of aberrations on the coherently combined fiber laser beams. Appl Opt 2008;47:3350–9.
[21] Architecture for diode high energy laser systems 〈http://www.darpa.mil/mto/programs/adhels/index.htm〉.
[22] Salem M, Shirai T, Dogariu A, Wolf E. Long-distance propagation of partially coherent beams through atmospheric turbulence. Opt Commun 2003;216: 261–5.