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Scintillation index of optical wave propagating in turbulent atmosphere*

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A concise expression of the scintillation index is proposed for a plane optical wave and a spherical optical wave both propagating in a turbulent atmosphere with a zero inner scale and a finite inner scale under an arbitrary fluctuation condition. The expression is based on both the results in the Rytov approximation under a weak fluctuation condition and the numerical results in a strong fluctuation regime. The maximum value of the scintillation index and its corresponding Rytov index are evaluated. These quantities are affected by the ratio of the turbulence inner scale to the Fresnel size.

Keywords: atmospheric optics, scintillation, atmospheric turbulence
PACC: 4225B, 9265

1. Introduction

Scintillation has long been a central topic of the research on light propagation in a turbulent atmosphere.[1] Though much effort has been devoted to theoretical analysis, experimental study and numerical simulation, the scintillation theory is established satisfactorily only in a weak fluctuation regime.[2] Some analytic asymptotic formulations were obtained for the scintillation index in a strong fluctuation regime, but they are not satisfactorily consistent and lacking in quantitative experimental verification.[3,4]

In many practical applications the light propagation condition is in the onset regime from weak to strong fluctuations. This regime is also called the strong focusing regime because the turbulent medium presents the most powerful focusing ability. Thus in this regime the light intensity will fluctuate to an extreme extent. It is very difficult to make any approximation for theoretical analysis in this case. Thus there has not yet existed a theoretical formulation on the scintillation in this regime. There is also a practical difficulty in experiment under the strong fluctuation condition, and only limited experiment events can present valuable results.[5] Some numerical simulations were performed, and quantitative results were obtained for several values of the Fresnel size and the turbulence inner size.[6−8] Unfortunately the numerical simulation results are not consistent with those obtained from the asymptotic formula in the strong fluctuation regime.

An explicit expression is necessary for the scintillation index in the onset regime in many applications. Andrews *et al*[9,10] proposed a scintillation theory based on heuristic reasoning. This theory is referred to as the modified Rytov approximation, and it is called the APHA scintillation theory in the present paper. Though the APHA theory is built for the scintillation problem, the modified spectrum introduced into this theory has been used also for phase fluctuation problems.[11,12] Whether the theoretical basis for scintillation is suitable for phase problem is still an open question.

The APHA theory for scintillation relies on the asymptotic formula under the strong fluctuation, and the latter is not fully supported both by numerical simulations and by measurements.[6,8,13] Also, in the case of a finite inner scale the expression is rather complicated and cannot be used easily.

A much simplified expression for the scintillation index under an arbitrary fluctuation condition with a finite inner scale is proposed in the present paper. The general expression is based on both the scintillation index under the weak fluctuation (the so called Rytov approximation result) and the numerical results in the strong fluctuation regime. Only one assumption is made that the general scintillation index as a function of the Rytov index has a unique analytic form for all fluctuation conditions.

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2. The APHA scintillation theory

The essence of the APHA theory is to construct a modified turbulence spectrum which depends both on the turbulence and on the light propagation condition. Two characteristic spatial wavenumbers κ_x and κ_y are synthesized for large and small scales respectively. The APHA theory in the weak fluctuation regime coincides with the Rytov approximation because the modified spectrum coincides with the true turbulence spectrum. The expressions of κ_x and κ_y are established through a comparison between the results deduced from the modified spectrum and the results obtained from the asymptotic formula in the saturation regime of Ref.[14]. There are some asymptotic results for the saturation regime in Ref.[4], which are different from those in Ref.[14]. Numerical results also show different behaviours in the strong regime.[6,8] Thus the reliability of the APHA theory is not unquestionable.

The APHA theory claims that the irradiance can be modelled as a modulation process in which small-scale (diffractive) fluctuations are multiplicatively modulated by large-scale (refractive) fluctuations. The irradiance is expressed as a product of two quantities x and y. They are related to the contributions of large and small turbulence scales. But it is difficult to figure out the physical meaning of the quantities x and y. They are surely not the amplitudes.

All assumptions made for APHA theory are equivalent to disregard of the contribution of the intermediate scale (about the Fresnel size) turbulence in the strong fluctuation regime. The thus modified spectrum is expressed as a filtered turbulence spectrum

$$\Phi_{\mathrm{m}}(\kappa) = \Phi_{\mathrm{n}}(\kappa)\left[G_x(\kappa) + G_y(\kappa)\right], \tag{1}$$

where $\Phi_{\mathrm{n}}(\kappa)$ is the turbulence spectrum, $G_x(\kappa)$ the low pass filter function, and $G_y(\kappa)$ the high pass filter function. For a Kolmogorov turbulence, $\Phi_{\mathrm{n}}(\kappa) = 0.033\,C_{\mathrm{n}}^2\kappa^{-11/3}$, $G_x(\kappa) = \exp(-\kappa^2/\kappa_x^2)$, and $G_y(\kappa) = \kappa^{11/3}/(\kappa^2+\kappa_y^2)^{11/6}$. The cutoffs κ_x and κ_y are chosen according to the Fresnel size l_{Fr} and the coherence length ρ_0. The filter functions and the modified spectra in the onset and strong fluctuation regime are plotted in Fig.1.

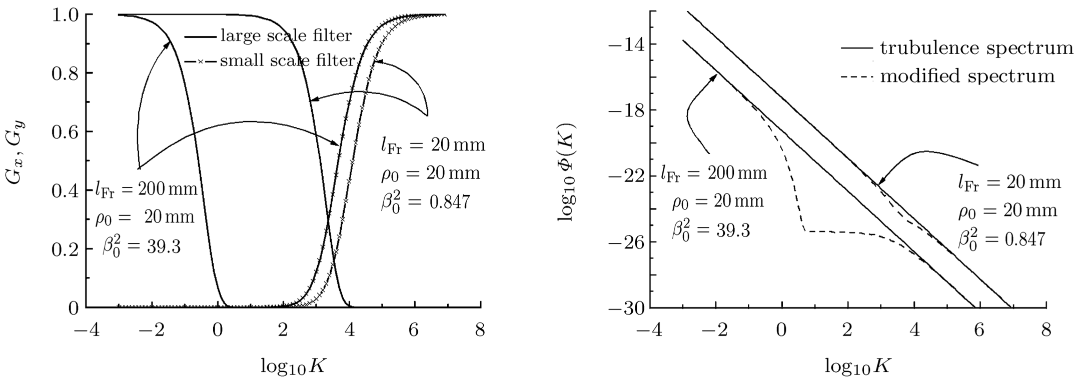

Fig.1. The filter functions and the modified spectra in the onset and strong fluctuation regimes.

So, the treatment is actually to divide a whole turbulence system with the inner and outer scales l_0 and L_0 into a couple of subsystems: one has the inner scale l_0 and the outer scale L_0', and the other has the inner scale l_0' and the outer scale L_0. These two effective inner and outer scales l_0' and L_0' are determined by the Fresnel size l_{Fr} and the coherence length ρ_0, i.e.

$$l_0' = l_{\mathrm{Fr}}\sqrt{1+(l_{\mathrm{Fr}}/\rho_0)^2}/\sqrt{3},$$

and

$$L_0' = \rho_0/\sqrt{1.7+3\,(\rho_0/l_{\mathrm{Fr}})^2}, \tag{2}$$

where, $l_{\mathrm{Fr}} = \sqrt{L/k}$, and k is the wavenumber, L the propagation length.

It is well known that the inner scale and the spectrum behaviour of the high frequency regime have an obvious effect on the scintillation. The way of selecting l_0' and L_0' will undoubtedly bring in some uncertainty of the results. At the present time there does not exist any experiment to check the theory about the dependence of the scintillation index on the turbulence inner scale and the Fresnel size.

The scintillation indices with a zero inner scale for a plane wave and a spherical wave obtained by the APHA theory are respectively:[10]

$$\beta_{\mathrm{I}}^2\,(\mathrm{pl}) = \exp\left\{\frac{0.49\beta_0^2}{\left(1+1.11\beta_0^{12/5}\right)^{7/6}} + \frac{0.51\beta_0^2}{\left(1+0.69\beta_0^{12/5}\right)^{5/6}}\right\} - 1\,, \tag{3a}$$

$$\beta_{\mathrm{I}}^2\,(\mathrm{sp}) = \exp\left\{\frac{0.49\beta_0^2}{\left(1+0.56\beta_0^{12/5}\right)^{7/6}} + \frac{0.51\beta_0^2}{\left(1+0.69\beta_0^{12/5}\right)^{5/6}}\right\} - 1\,, \tag{3b}$$

where the Rytov index $\beta_0^2 = \alpha C_{\mathrm{n}}^2 k^{7/6} L^{11/6}$, $\alpha = 1.23$ for the plane wave, and 0.5 for the spherical wave.

The expressions of the scintillation indices with a finite inner scale are rather complicated, and they are not listed here.

3. Generalized scintillation theory

The scintillation index under weak fluctuation condition is well established in the Rytov approximation as $\beta_{\mathrm{I}}^2 = \beta_0^2 \quad (\beta_0^2 \ll 1)$. Let $\beta_\infty^2 = \beta_{\mathrm{I}}^2, (\beta_0^2 \gg 1)$ represent the scintillation index under the strong fluctuation condition. Assume that the general scintillation index as a function of the Rytov index has a unique analytic form for all fluctuation conditions. Then the general expression of scintillation index under any fluctuation condition should be related to β_0^2 and β_∞^2 as follows:[15]

$$1/\beta_{\mathrm{I}}^2 = 1/\beta_0^2 + 1/\beta_\infty^2. \tag{4}$$

Thus we have

$$\beta_{\mathrm{I}}^2 = \beta_0^2\beta_\infty^2/(\beta_0^2 + \beta_\infty^2). \tag{5}$$

Both analytic and numerical studies indicate that the scintillation index β_∞^2 under the condition of strong fluctuation presents a behaviour as

$$\beta_\infty^2 = 1 + a(\beta_0^2)^{-b}. \tag{6}$$

Therefore we have the generalized scintillation index

$$\beta_{\mathrm{I}}^2 = \frac{a\beta_0^2 + \left(\beta_0^2\right)^{1+b}}{a + \left(\beta_0^2\right)^b + \left(\beta_0^2\right)^{1+b}}. \tag{7}$$

This designed function accords with the Rytov approximation and the asymptotic formula in both ends. It surely has a maximum in the strong focusing regime, and it generally represents the characteristics of the scintillation behaviour.

The analytic asymptotic formulae for a plane wave and a spherical wave used in the APHA theory are:[9]

$$\beta_\infty^2(\mathrm{pl}) = 1 + 0.86(\beta_0^2)^{-2/5}, \tag{8a}$$

and

$$\beta_\infty^2(\mathrm{sp}) = 1 + 1.9(\beta_0^2)^{-2/5}. \tag{8b}$$

The generalized scintillation indices for a plane wave and a spherical wave in this case are respectively

$$\beta_{\mathrm{I}}^2(\mathrm{pl}) = \frac{\beta_0^2 + 0.86(\beta_0^2)^{3/5}}{\beta_0^2 + 1 + 0.86(\beta_0^2)^{-2/5}}, \tag{9a}$$

and

$$\beta_{\mathrm{I}}^2(\mathrm{sp}) = \frac{\beta_0^2 + 1.9(\beta_0^2)^{3/5}}{\beta_0^2 + 1 + 1.9(\beta_0^2)^{-2/5}}. \tag{9b}$$

Analytic asymptotic formulae could also be built based on the numerical simulation result (zero inner scale) as follows:[8]

$$\beta_\infty^2(\mathrm{pl}) = 1 + 1.8216(\beta_0^2)^{-1/2}, \tag{10a}$$

and

$$\beta_\infty^2(\mathrm{sp}) = 1 + 8.3567(\beta_0^2)^{-0.65}. \tag{10b}$$

The generalized scintillation indices for a plane wave and a spherical wave in this case are respectively

$$\beta_{\mathrm{I}}^2(\mathrm{pl}) = \frac{\beta_0^2 + 1.8216(\beta_0^2)^{1/2}}{\beta_0^2 + 1 + 1.8216(\beta_0^2)^{-1/2}}, \tag{11a}$$

and

$$\beta_{\mathrm{I}}^2(\mathrm{sp}) = \frac{\beta_0^2 + 8.3567(\beta_0^2)^{0.35}}{\beta_0^2 + 1 + 8.3567(\beta_0^2)^{-0.65}}. \tag{11b}$$

For a finite inner scale case, the asymptotic formulae based on the numerical simulation result depend on l_0/l_{Fr}, the ratio of inner scale to Fresnel size,[8] i.e.

$$\beta_\infty^2(\mathrm{pl}) = 1 + 2.2085(l_0/l_{\mathrm{Fr}})^{0.45}(\beta_0^2)^{-0.33}, \tag{12a}$$

and

$$\beta_\infty^2(\mathrm{sp}) = 1 + 20.923(l_0/l_{\mathrm{Fr}})^{0.8}(\beta_0^2)^{-0.45}. \tag{12b}$$

The generalized scintillation indices for a plane wave and a spherical wave in this case are respectively

$$\beta_{\rm I}^2({\rm pl}) = \frac{\beta_0^2\left(1+2.2085(l_0/l_{\rm Fr})^{0.45}(\beta_0^2)^{-0.33}\right)}{\beta_0^2+1+2.2085(l_0/l_{\rm Fr})^{0.45}(\beta_0^2)^{-0.33}}, \quad (13a)$$

and

$$\beta_{\rm I}^2({\rm sp}) = \frac{\beta_0^2\left(1+20.923(l_0/l_{\rm Fr})^{0.8}(\beta_0^2)^{-0.45}\right)}{\beta_0^2+1+20.923(l_0/l_{\rm Fr})^{0.8}(\beta_0^2)^{-0.45}}. \quad (13b)$$

The generalized scintillation indices each as a function of the Rytov index in the cases of zero and non-zero inner scales for plane and spherical waves are plotted in Figs.2 and 3 respectively. For the non-zero inner scale case, the scintillation indices are evaluated, with values of $l_0/l_{\rm Fr}$ being 0.2, 0.4, 0.6, 0.8 and 1.0. They are represented by solid lines in the figures, with the value of $l_0/l_{\rm Fr}$ increasing from bottom to top. For the zero inner scale case three results (3), (9), and (11) are all plotted together for comparison. These three results for the zero inner scale case are clearly different from each other. Among them, only the analytical result from expression (9) is more consistent with that for the non-zero inner scale case and the numerical result from expression (11) is the most strange.

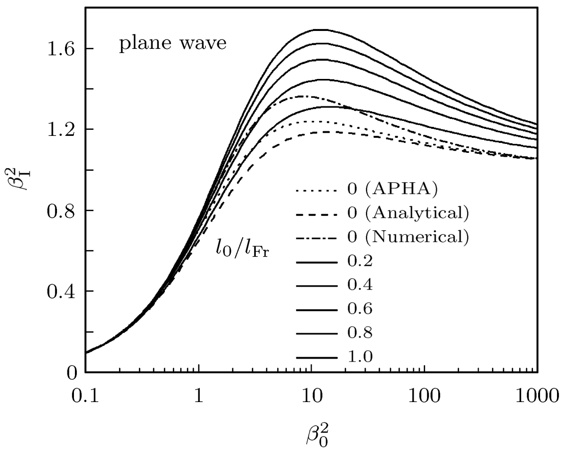

Fig.2. The general scintillation index of a plane wave as a function of Rytov index for several different values of $l_0/l_{\rm Fr}$, where solid curves are for non-zero inner scale cases with the value of $l_0/l_{\rm Fr}$ increasing from bottom to top.

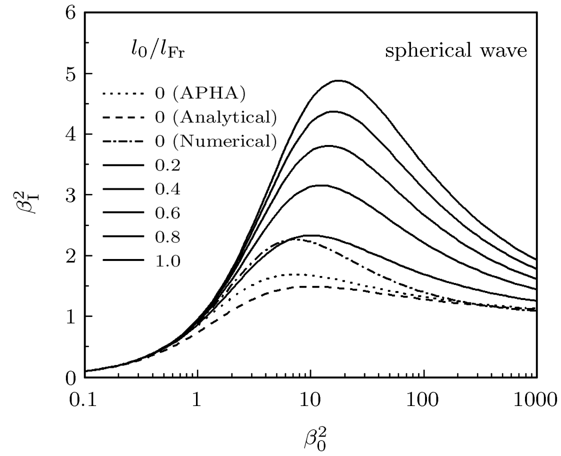

Fig.3. The general scintillation index of a spherical wave as a function of Rytov index for several different values of $l_0/l_{\rm Fr}$, where solid curves are for non-zero inner scale case with the value of $l_0/l_{\rm Fr}$ increasing from bottom to top.

4. Maximum scintillation index

From the general formula of scintillation index (expression (7)) we can find out the Rytov index at which the scintillation index reaches its maximum value. Let the derivative of expression (7) be zero, then we will obtain the following equation:

$$\left(\beta_0^2\right)^{2b} - ab\left(\beta_0^2\right)^{b+1} + 2a\left(\beta_0^2\right)^{b} + a^2 = 0. \quad (14)$$

We can further obtain

$$\left(\beta_0^2\right)^{b} - \sqrt{ab}\left(\beta_0^2\right)^{(b+1)/2} + a = 0. \quad (15)$$

Thus we can determine the relation between the Rytov index corresponding to the maximum scintillation index and constants a and b. For the finite inner scale case, constant a depends on $l_0/l_{\rm Fr}$. Therefore we can determine the relation between the Rytov index corresponding to the maximum scintillation index and $l_0/l_{\rm Fr}$.

When we obtain the Rytov index corresponding to the maximum scintillation index, we can further acquire the scintillation maximum index itself through expression (7). Thus we can also determine the maximum scintillation index as a function of $l_0/l_{\rm Fr}$.

As Eq.(15) is a nonlinear equation, it is difficult to find an analytical solution. But we can obtain the solution numerically. The numerical solutions in zero and non-zero inner scale cases for a plane wave and a spherical wave are carried out. For the non-zero inner scale case five values of $l_0/l_{\rm Fr}$ from 0.2 to 1 are evaluated. The maximum scintillation index (5$^{\rm th}$ column) and the corresponding Rytov index (the 4$^{\rm th}$ column) are listed in Tables 1 and 2 for a plane wave and a spherical wave, respectively.

Table 1. The maximum scintillation indices and corresponding Rytov indices for a plane wave.

l_0/l_{Fr}	a (Eq.(12a))	b (Eq.(12a))	β_0^2	β_{I}^2	a (Eq.(16))	b (Eq.(16))	β_0^2	β_{I}^2
0 (analytical)	0.86	0.4	14.1309	1.1889				
0 (numerical)	1.8216	0.5	8.4445	1.3641				
0 (extrapolated)			15.6732	1.1677	0.7850	0.365	15.8283	1.1898
0.2	1.0704	0.33	14.2080	1.3123	1.1287	0.350	13.2768	1.3126
0.4	1.4623	0.33	12.7764	1.4462	1.4724	0.335	12.5880	1.4434
0.6	1.7550	0.33	12.3387	1.5448	1.1816	0.320	12.6454	1.5806
0.8	1.9975	0.33	12.1751	1.6252	2.1599	0.305	13.1159	1.7242
1.0	2.2085	0.33	12.1247	1.6942	2.5036	0.290	13.8765	1.8748

Table 2. The maximum scintillation indices and corresponding Rytov indices for a spherical wave.

l_0/l_{Fr}	a (Eq.(12b))	b (Eq.(12b))	β_0^2	β_{I}^2	a (Eq.(17a))	b (Eq.(17b))	β_0^2	β_{I}^2
0 (analytical)	1.9	0.4	10.2081	1.4940				
0 (numerical)	8.3567	0.65	7.2833	2.2705				
0 (extrapolated)			7.5676	1.4575	0.38435	0.385	30.7260	1.0646
0.2	5.7736	0.45	10.0708	2.3363	5.85033	0.471	9.5833	2.2951
0.4	10.0525	0.45	12.3401	3.1583	11.9104	0.497	11.6657	3.2541
0.6	13.9042	0.45	14.3054	3.8133	16.4489	0.463	14.9385	4.1277
0.8	17.5023	0.45	16.0431	4.3775	17.3450	0.369	21.1509	5.0458
1.0	20.9230	0.45	17.5172	4.8821	12.4979	0.215	36.8750	5.7086

For zero inner scale case the results based on the analytical asymptotic and numerical strong fluctuation behaviours are listed in the tables. Constants a and b are quite different for these two treatments, consequently, the maximum scintillation index and relevant Rytov index are also different. The extrapolated values from the non-zero inner scale case are also listed for comparison. For the plane wave, the extrapolated results are quite consistent with the analytical ones. But for the spherical wave, the extrapolated Rytov index position is consistent with the numerical one, and the extrapolated maximum scintillation index is similar to the analytical one. We note that the asymptotic behaviour of the plane wave has been thoroughly investigated and verified, and the spherical counterpart is not widely used. As for the numerical results, the strong fluctuation behaviour of the zero-inner scale scintillation is quite different from that for the non-zero inner scale case. The numerically obtained result for the zero inner scale is not consistent with the extrapolated one from the result for the non-zero inner scale.

We notice that the maximum scintillation index predicted above is not so large as the experimental and numerical results in the case of high value of l_0/l_{Fr}. The constants a and b are chosen according to Ref.[8], and they are obtained for all non-zero inner scale data by using a most simple fitting way, with constant a other than b chosen to be dependent on ratio l_0/l_{Fr}. Actually both constants a and b are dependent on ratio l_0/l_{Fr} according to the numerical results, especially for a spherical wave. To further check the maximum scintillation index behaviour the two constants each will be fitted more accurately as a function of ratio l_0/l_{Fr}.

For a plane wave, the constants a and b can each be fitted as a linear function of ratio l_0/l_{Fr}, given as

$$a = 0.7850 + 1.7186\,(l_0/l_{\mathrm{Fr}})\,,$$

and

$$b = 0.365 - 0.075\,(l_0/l_{\mathrm{Fr}})\,. \tag{16}$$

For a spherical wave, the constants a and b each should be better fitted as a polynomial function of l_0/l_{Fr}, expressed as

$$\begin{aligned} a = \;& 0.38435 + 22.3184\,(l_0/l_{\mathrm{Fr}}) \\ & + 33.8733\,(l_0/l_{\mathrm{Fr}})^2 - 44.0782\,(l_0/l_{\mathrm{Fr}})^3\,, \end{aligned} \tag{17a}$$

and

$$b = \; 0.385 + 0.58\,(l_0/l_{\mathrm{Fr}}) - 0.75\,(l_0/l_{\mathrm{Fr}})^2\,. \tag{17b}$$

These constants will be used to find the maximum scintillation index. The relevant results are listed in the rightmost 4 columns in Tables 1 and 2. For $l_0/l_{\rm Fr} = 0.8$ and 1, the newly obtained maximum scintillation indices are 5.05 and 5.71 instead of 4.38 and 4.88. These results are more consistent with the experimental and numerical simulation data. Thus it is important to accurately obtain the asymptotic behaviour of the scintillation index under the strong fluctuation condition for the application of the general scintillation theory.

The effects of $l_0/l_{\rm Fr}$ on the maximum scintillation index and the corresponding Rytov index are plotted in Figs.4 and 5 based on the data in the rightmost 2 columns in Tables 1 and 2. As for the Rytov index the behaviours of a plane wave and a spherical wave are quite different. For a plane wave, the Rytov index varies rather smoothly with $l_0/l_{\rm Fr}$. For a spherical wave, the Rytov index position varies rather abruptly with $l_0/l_{\rm Fr}$, and presents a minimum at $l_0/l_{\rm Fr} = 0.2$.

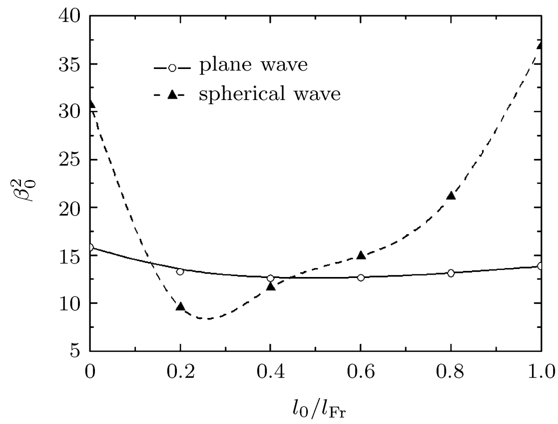

Fig.4. The Rytov indices corresponding to the maximum scintillation index as a function of $l_0/l_{\rm Fr}$ for a plane wave and a spherical wave.

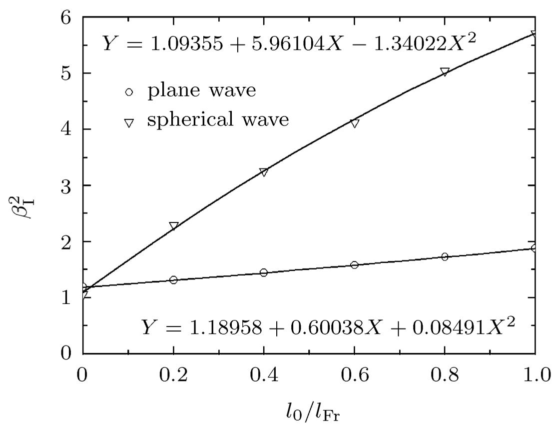

Fig.5. The maximum scintillation indices as a function of $l_0/l_{\rm Fr}$ for a plane wave and a spherical wave.

The behaviours of the maximum values of the scintillation index of a plane wave and a spherical wave are also quite different. For a plane wave, the maximum scintillation index increases very slowly with ratio $l_0/l_{\rm Fr}$ increasing from 0.2 to 1. For a spherical wave, the maximum scintillation index increases significantly from about 1 to 5.7 with the value of $l_0/l_{\rm Fr}$ increasing. Polynomials for fitting the results are as follows:

$$\beta_{\rm I}^2({\rm pl}) = 1.19 + 0.60\,(l_0/l_{\rm Fr}) + 0.085\,(l_0/l_{\rm Fr})^2\,, \quad (18a)$$

and

$$\beta_{\rm I}^2({\rm sp}) = 1.09 + 5.96\,(l_0/l_{\rm Fr}) - 1.34\,(l_0/l_{\rm Fr})^2\,. \quad (18b)$$

5. Discussion

The general scintillation theory proposed here depends only on the Rytov index and two constants which depict asymptotic behaviours. Thus the general expression of the scintillation index is simple in formulation. A predicted maximum scintillation index appears in the strong focusing regime. This result is consistent with experimental result and similar to that predicted by the APHA theory. It must be emphasized that this general scintillation theory is valid only for the scintillation index.

With the current theory, the maximum scintillation index and its corresponding Rytov index each are well predicted to be a function of those constants. In the non-zero inner scale case they are functions of ratio of the inner scale and Fresnel size. This will serve as an object for verifying the current theory with further detailed quantitative experimental and numerical studies.

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