JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 92, NO. A3, PAGES 2565–2569, MARCH 1, 1987

On Calculating Ionospheric Conductances From the Flux and Energy of Precipitating Electrons

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Auroral zone conductances can be estimated from the energy flux and average energy of precipitating electrons. These estimates are based on the assumption that the conductances produced by the electrons are very similar to those produced by electrons with Maxwellian energy distributions having the same energy flux and average energy. There has been some confusion in the application of this method because for a Maxwellian the average energy is twice the characteristic energy or temperature. We present revised expressions that relate height-integrated Hall and Pedersen conductance to the flux and average energy of a Maxwellian. We show that the accuracy of this method depends on the minimum and maximum energy within which the distribution is integrated to get the energy flux and average energy. We also confirm that the conductances produced by some of the more common auroral spectral distributions are similar to those produced by a Maxwellian with the same average energy and energy flux. The application of these results is demonstrated using precipitating electron measurements made by the Hilat satellite during a pass over Greenland.

INTRODUCTION

In the past several years there has been considerable effort devoted to producing maps of auroral zone conductances from global surveys of energetic electron precipitation. Such global surveys have been carried out using data from polar orbiting satellites such as ISIS [*Wallis and Budzinski*, 1981], AE-C [*Spiro et al.*, 1982], DMSP (D. Hardy, private communication, 1986) and NOAA (D. S. Evans, private communication, 1986). Results of studies reported by *Vickrey et al.* [1981] and *Vondrak and Robinson* [1985] have shown that conductances can be accurately computed from the electron spectral distribution. However, because the statistical studies involve processing large amounts of data, it is sometimes more efficient to use simple expressions that relate the energy flux and average energy to the height-integrated Hall and Pedersen conductances. In previous studies this relationship has been established by assuming the shape of the electron energy distribution and using numerical codes to calculate the resulting ionization profile and ionospheric conductances. The results of *Vickrey et al.* [1981] have been used in several auroral conductivity models [*Spiro et al.*, 1982; *Simons et al.*, 1985; D. Hardy, private communication, 1986]. The relations presented by *Vickrey et al.* [1981] assumed electron energy distributions given by

$$F(E) = AE \exp(-E/E_0) \tag{1}$$

where E is the electron energy, E_0 is the characteristic energy and A is a constant that is proportional to the energy flux. Because most auroral electron energy distributions are nearly Maxwellian in shape, the resulting conductances are good estimates of those actually produced. However, there is some confusion in the literature regarding the interpretation of the average energy in the application of the results of *Vickrey et al.* [1981]. The purpose of this paper is to clear up some of the confusion regarding the meaning of average energy and its use in estimating ionospheric conductances. After carefully defining the average energy we present revised expressions relating the energy flux and average energy of a Maxwellian to the height-integrated Hall and Pedersen conductance. We also show that the correct application of these relations requires the proper choice of the energy limits within which the electron distribution is integrated to get energy flux and average energy. By calculating conductances produced by several other types of distributions we show that the Maxwellian relations are valid for most common auroral electron spectra. Electron spectral measurements made by the Hilat satellite are used to demonstrate the accuracy of these relations when properly applied.

RELATIONS BETWEEN AVERAGE ENERGY AND CONDUCTANCE

In modeling conductances produced by precipitating electrons it is customary to relate the conductances to the average energy of precipitating particles. The average energy $\bar{E}$ is computed from measurements of the fluxes of precipitating electrons according to

$$\bar{E} = \frac{\int_{E_{min}}^{E_{max}} EF(E)\,dE}{\int_{E_{min}}^{E_{max}} F(E)\,dE} \tag{2}$$

where E_{min} and E_{max} are the minimum and maximum energies measured by the detector and $F(E)$ is the differential electron flux. The conductances given by *Vickrey et al.* [1981] are given as a function of the characteristic energy which is E_0 in equation (1) and is equivalent to the temperature of the Maxwellian. For a Maxwellian the average energy computed according to equation (2) is twice the characteristic energy. In contrast, for an exponential power law distribution of the form $F(E) = A \exp(-E/E_0)$ the average energy is equal to the characteristic energy E_0. Thus, the average energy is a parameter that is uniquely defined by the distribution function according to equation (2), whereas the relation between the characteristic energy and average energy depends on the shape of the distribution. To avoid this ambiguity, we present below

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Paper number 6A8586.
0148-0227/87/006A-8586$02.00

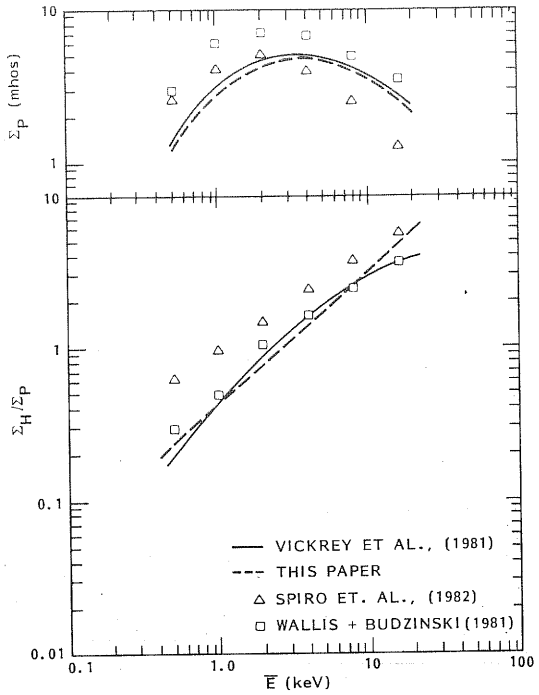

Fig. 1. Comparison of relations between conductances and the average energy of a Maxwellian distribution with an energy flux of 1 ergs/cm^2 s. The results of *Vickrey et al.* [1981] represent those obtained using an energy deposition code. The results shown by dashed lines are those given by equations (3) and (4) of this paper.

expressions relating Hall and Pedersen conductances to the average energy and energy flux of the electrons:

$$\Sigma_P = \frac{40\bar{E}}{16 + \bar{E}^2}\,\Phi_E{}^{1/2} \tag{3}$$

$$\frac{\Sigma_H}{\Sigma_P} = 0.45(\bar{E})^{0.85} \tag{4}$$

where Σ_P and Σ_H are the Pedersen and Hall conductances, respectively, $\bar{E}$ is the average energy in keV and Φ_E is the energy flux in ergs/cm^2 s. It should be emphasized that these expressions represent fits to the calculated values and therefore are not exact. The solid line in Figure 1 shows the results presented by *Vickrey et al.* [1981] plotted as a function of average energy instead of characteristic energy. The calculations are based on energy deposition functions derived by *Rees* [1963]. This method for computing the height profile of electron density between 80 and 200 km altitude has been validated by *Vondrak and Robinson* [1985]. The *Banks and Kockarts* [1973] 1000° exosphere neutral atmosphere model was used with an ion-neutral collision frequency given by $3.75 \times 10^{-10}N$ s^{-1}, where N is the neutral number density in cm^{-3}. The dashed lines in Figure 1 indicate the fits to these values given by equations (3) and (4). The fitted values are within 20% of the actual values below about 10 keV. The triangles in Figure 1 show the fits to the results of *Vickrey et al.* [1981] presented by *Spiro et al.* [1982]. The difference in the fits results from the assumption made by *Spiro et al.* [1982] that the characteristic energy and average energy are equal. This assumption, which is not consistent with the assumed Maxwellian distributions used by *Vickrey et al.* [1981], leads to Pedersen conductances that differ from the actual values by as much as a factor of 2.

Figure 1 also includes conductances computed by *Wallis and Budzinski* [1981]. Although the Hall to Pedersen ratio agrees well with our values, the Pedersen conductances computed by *Wallis and Budzinski* [1981] are systematically higher. *Wallis and Budzinski* [1981] used essentially the same method as that used by *Vickrey et al.* [1981] to derive the conductances from electron fluxes so that the source of this systematic difference is unknown. However, as mentioned above, the code used by *Vickrey et al.* [1981] has been validated using simultaneous electron spectrometer data from the AE-C satellite and ionization measurements made by the Chatanika incoherent scatter radar [*Vondrak and Robinson*, 1985].

Calculation of Average Energy

The average energy as defined in equation (2) depends on the values used for E_{min} and E_{max}. Errors can result in the use of equations (3) and (4) if either E_{min} or E_{max} is too low. Let us first consider the situation in which E_{max} is such that a significant number of high-energy electrons are excluded from the calculation. Most electron spectrometers on satellites have upper energy limits of 20–30 keV. When the average energy of the electrons is high a correction factor should be applied to

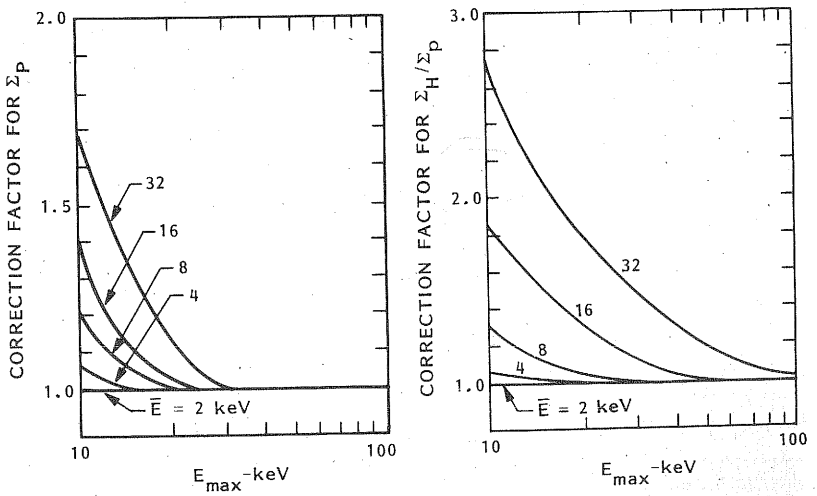

Fig. 2. Correction factor to be applied to the results given by (3) and (4) when the spectrum is measured below an energy E_{max}. Curves are given for five different values of the average energy.

the calculations to account for the undetected electrons. We have calculated correction factors to the Hall and Pedersen conductance as a function of E_{max} by assuming Maxwellian energy distributions of varying average energy. The results are shown in Figure 2. The curves are given for average energies of 2, 4, 8, 16 and 32 keV. As an example, for a detector that measures electrons only up to 20 keV, correction factors of 1.03 and 1.3 must be applied to the Pedersen conductance and Hall to Pedersen ratio, respectively, if the average energy is 16 keV. The Hall to Pedersen ratio is affected more because the higher energy electrons affect the Hall conductance to a greater extent than the Pedersen conductance. These corrections are only approximate, since the value of the average energy itself may be in error owing to the lack of information about the high-energy portion of the spectrum.

Additional errors arise in the application of equations (3) and (4) when E_{min} is too low. Because the electron energy distributions used in deriving the relations between average energy and conductance are Maxwellians, the fluxes decrease monotonically with decreasing energy for energies less than E_0. However, for energies below about 500 eV typical auroral electron distributions are characterized by fluxes considerably higher than that given by a Maxwellian. This is because the low energy electrons consist of not only the primary auroral electrons which are nearly Maxwellian, but also the secondaries that are produced on interaction of the primary electrons with the atmosphere. Fortunately, these low energy electrons contribute little to the height-integrated conductance because they deposit their energy at altitudes where the collision frequency is too low to sustain currents perpendicular to the magnetic field. Thus, it is sufficient to use as a lower limit of integration the energy at which secondary electrons begin to dominate over the primary electrons. The numerical results of *Banks et al.* [1974] indicate that secondary fluxes become important at energies below about 500 eV for primary distributions with average energies below 10 keV. Thus, 500 eV is a convenient energy at which to terminate the integration in equation (2).

To illustrate the importance of the correct choice of E_{min} we show in Figure 3 two electron spectra measured by the Hilat satellite during a pass over a series of morning sector auroral arcs on November 17, 1985. A calculation of the electron density profile and resulting conductances using the spectrum shown by the solid line was performed using the energy deposition code described above. The height-integrated conductances were $\Sigma_P = 6.2$ mhos and $\Sigma_H = 11.5$ mhos. When the entire spectrum is used to compute average energy and energy flux the result is $\bar{E} = 0.2$ keV and $\Phi_E = 3.1$ ergs/cm^2-s. When used in equations (3) and (4), these give values for Pedersen and Hall conductance of 0.88 and 0.10 mhos, respectively. If the integration is terminated at 500 eV the average energy is 4.9 keV and the energy flux is 1.9 ergs/cm^2-s. These values yield Pedersen and Hall conductances of 6.8 and 11.6 mhos, respectively, which are almost equal to the true values.

The spectrum shown by the solid line in Figure 3 represents a rather extreme example in that the fluxes of electrons at low energies were unusually intense. For comparison the dashed curve shows an electron spectrum measured in a different location within the same auroral form. The flux at about 100 eV is less by more than 2 orders of magnitude than that given by the solid curve. Because of the high variability in the low-energy portion of auroral distributions, no universal correction can be applied to the conductances when the entire spectrum is used. The error resulting from using too small a value for E_{min} for an arbitrary spectrum depends on the extent to which the fluxes below 500eV differ from a Maxwellian. Although the magnitude of these enhancements may not be as pronounced as those shown in Figure 3, any difference between the actual spectrum and a Maxwellian will distort the conductance values calculated from equations (3) and (4).

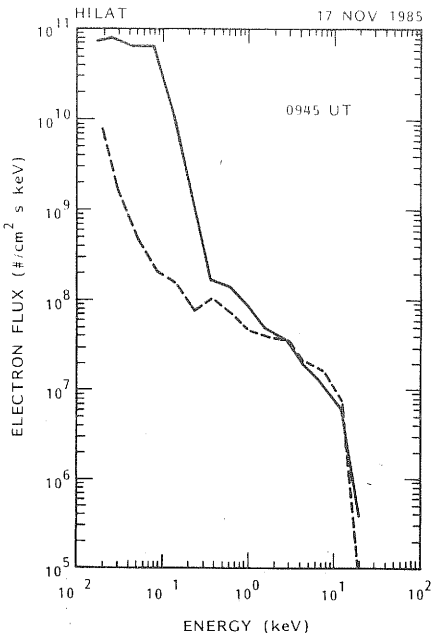

Fig. 3. Two electron distributions measured by Hilat on November 17, 1985, over multiple morning sector auroral arcs.

CONDUCTANCES FOR OTHER SPECTRAL DISTRIBUTIONS

In applying equations (3) and (4) to an arbitrary spectral distribution the assumption is made that the spectrum is sufficiently similar to a Maxwellian that the resulting conductances will not differ from those computed by *Vickrey et al.* [1981]. It is of interest to estimate the magnitude of the error produced when these equations are used for spectra of other shapes. We performed this assessment by computing conductances produced by monoenergetic, exponential and Maxwellian spectra and intercomparing the results. Two types of Maxwellian spectra were used. The first type is given by equation (1). For the second type we used a 1-keV Maxwellian that has been accelerated through a parallel potential of varying voltages. When a Maxwellian is accelerated through a potential V_0, the result is a spectrum with no electrons below the energy V_0. However, the low-energy portion is filled in by backscattered and secondary electrons [*Evans*, 1974]. To estimate this component we used analytic functions derived by *Davidson* [1986]. The flux of low-energy electrons computed in this manner is somewhat lower than those estimated by *Evans* [1974], primarily because it neglects those upward moving electrons that are reflected back into the ionosphere from the bottom of the potential. However, the resulting spectrum resembles those typically measured within discrete auroral forms. To compare conductances from these various distributions we plotted the values as a function of the average energy. For the exponential spectrum given by $F(E) = A \exp(-E/E_0)$ the average energy is E_0. As stated earlier, for the

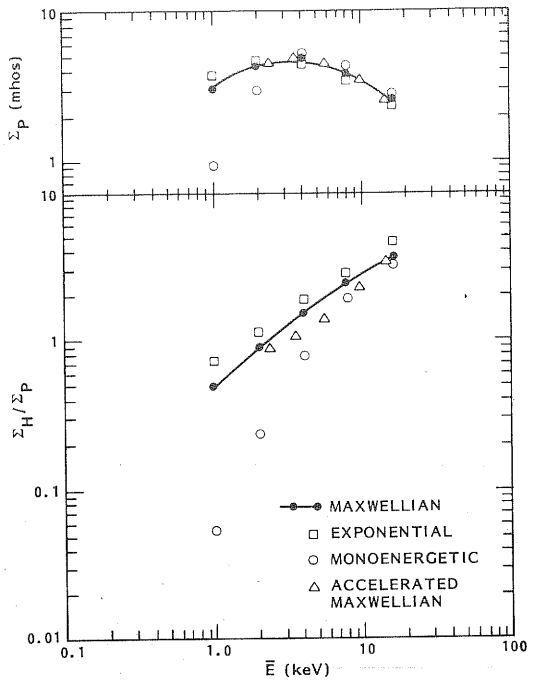

Fig. 4. Modelled conductances for Maxwellian, monoenergetic, exponential and accelerated Maxwellian electron distributions.

Maxwellian the average energy is twice E_0. For the accelerated Maxwellian with backscatter we computed the average energy from equation (1) using an E_{min} of 0.5 keV. All conductances, were normalized to a total energy flux of 1 erg/cm^2 s. The results of these calculations are shown in Figure 4. For average energies greater than about 4 keV the Pedersen conductances for the four different types of spectra agree to within 10%. For an average energy of 1 keV the monoenergetic spectrum produces a much lower Pedersen conductance than the other three because at these low energies most of the ionization produced by monoenergetic electrons is above 200 km altitude. Similarly, for the Hall to Pedersen conductance ratio the Maxwellian, exponential spectra and accelerated Maxwellian agree to within about 25% for average energies greater than a few keV. The monoenergetic spectrum produces Hall to Pedersen ratios that differ from the others by as much as an order of magnitude at the lower energies.

The results shown above indicate that the assumption of a Maxwellian spectrum yields conductivities that are within about 25% of the actual values for the most common types of auroral distributions. If the spectrum is monoenergetic much greater errors result. However, the monoenergetic spectrum is perhaps the most unphysical of the three. Because of the similarity in the conductances for the distributions examined here we expect that equations (3) and (4) are applicable to a variety of different electron spectra.

Example of Conductance Calculations

As an example of the concepts discussed above we show in Figure 5 conductances computed from Hilat satellite data obtained during a pass over Sondre Stromfjord, Greenland, at 0945 UT, November 17, 1985. The pass was part of a coordinated experiment involving Hilat and the Sondrestrom incoherent scatter radar. At the time of the pass, multiple auroral arcs were observed from the ground at the radar site. The electron spectrometer on Hilat measures fluxes from 0.02 keV to 20 keV in 16 channels. These fluxes were used in our energy deposition code to compute height-integrated conductivities which are shown by the solid lines in Figure 5. The solid dots show the conductances computed using equations (3) and (4) and a minimum energy of 0.6 keV. The dashed line shows the conductances derived if all energies are included in the calculation of energy flux and average energy. The solid and dashed arrows in the figure show the locations at which the solid and dashed spectra in Figure 3 were obtained. Note that the error associated with the dashed spectrum is negligible because of the absence of enhanced low energy electron fluxes. The triangles in Figure 5 show conductances computed using simultaneous and coincident measurements of electron density obtained by the Sondrestrom radar. The measurements agree well with the calculated values for the two points at 72° and 74° invariant latitude. At 75° the radar measured a higher Pedersen conductance, probably owing to spatial or temporal variations in the precipitating electron flux. The results in Figure 5 show that there is good agreement between the conductances measured by the radar, those inferred from equa-

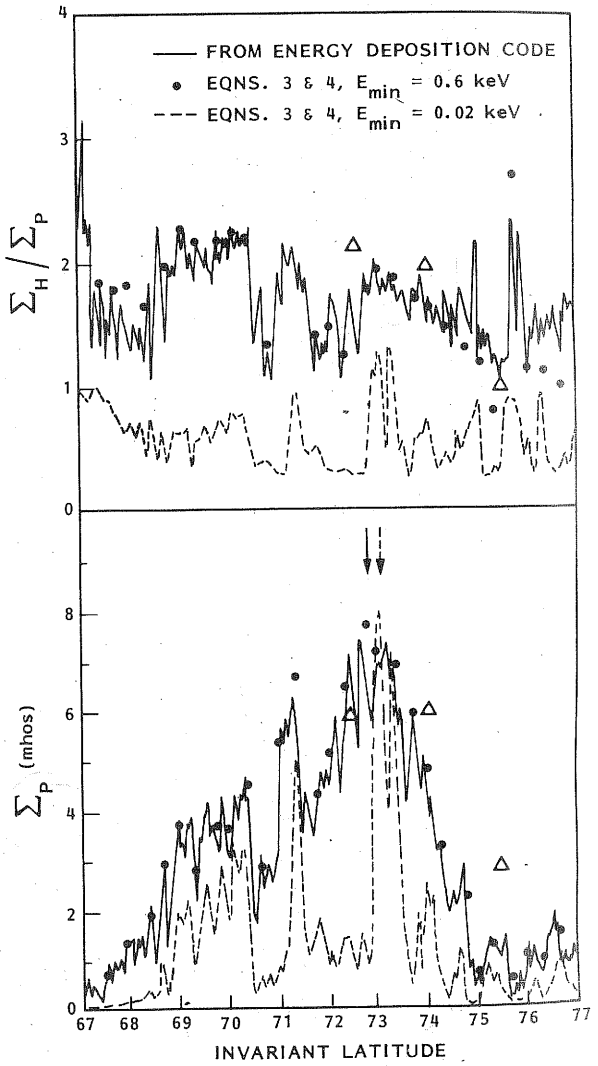

Fig. 5. Conductances produced by electrons measured by Hilat during the November 17 pass over multiple morning sector auroral arcs. Arrows give the locations at which the spectra in Figure 3 were measured. Triangles show the conductances inferred from simultaneous and coincident Sondrestrom radar measurements.

tions (3) and (4) and those calculated from the energy deposition code. Many of the spectra measured during this pass were similar to those shown in Figure 3. Although these spectra are not identical to any of the types used in Figure 4, the resulting conductances are still in good agreement with those calculated for a Maxwellian with the same average energy and energy flux. This further demonstrates the applicability of these results to a wide variety of auroral electron fluxes.

CONCLUSIONS

We have presented expressions relating the flux and average energy of precipitating electrons to the Hall and Pedersen conductances produced from the resulting ionization profile. By relating the conductances to the average energy defined according to equation (2), we have eliminated the ambiguity and confusion created in using the characteristic energy as an ordering parameter. In addition, we have shown that the conductances produced by Maxwellian electron distributions are good approximations to those produced by arbitrary distributions provided the arbitrary spectra are compared to Maxwellians with the same average energy. In computing the average energy of an arbitrary electron distribution it is important to choose the limits of integration carefully. If the spectra are only known to some maximum energy, correction factors may have to be applied to account for the electrons at higher energies. The minimum energy used in the integration should not be less than about 500 eV because electrons with lower energies do not contribute to the conductance and these may differ substantially from a Maxwellian distribution.

Acknowledgments. This work was supported at Lockheed by National Science Foundation grant ATM-8508129 and NASA contract NAS5-28714 and at SRI by DNA contract DNA001-85-C-0062.

The Editor thanks R. W. Spiro and another referee for their assistance in evaluating this paper.

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(Received June 10, 1986;
revised November 18, 1986;
accepted December 2, 1986.)