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Imperfection of flux pinning classification based on the pinning center size

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Abstract

Magnetic flux pinning classification based on the pinning center size is found to be imperfect because of the proximity effect of superconducting state. The application of this idea to the Dew-Hughes model shows that the eight flux pinning functions in it are linearly dependent, which implies that some of these pinning mechanisms are not the primal ones, but a combination of others. Based on the physical considerations, 'core, normal, surface pinning' and 'core, normal, point pinning' are abandoned. A pinning function that is related to the self-field critical current is induced. Then seven linearly independent pinning functions are picked out to represent the seven primal pinning mechanisms, respectively. The application of these seven functions to the experimental data shows that they contribute to a better general description of the complex mechanism of the pinning of the superconductors.

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1. Introduction

The pinning, or the interaction, of the flux-line lattice with various crystal imperfections in type-II superconductors is responsible for the existence of a critical current density J_c, usually defined as

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the current density at which an arbitrarily small voltage is observed. Flux-line interaction with crystal imperfections, or pinning centers, because the superconducting properties of the latter are different from those of the bulk of the superconductor. The strength of the interaction is a function of the magnitude of this different. The difference may be small, and manifest itself as a different in critical temperature, critical field, or Ginsburg–Landau parameter κ. The difference may be large as is the case when the pinning center is non-superconducting. Of all the various possibility, only two are believed responsible for flux-line in the majority of superconducting materials [1]: (i) small differences in κ, arising from changes in the normal state resistivity due to composition fluctuations, or non-uniform distributions of dislocations, (ii) non-superconducting particles, which may be normal metal, insulator or void. The two types of pinning centers give rise to what will subsequently be referred to as '$\Delta\kappa$ pinning' and 'normal pinning' by the superconducting nature of the pinning centers. The pinning centers are further classified as point pins, line pins, surface pins and volume pins by the number of the pinning centers' dimensions that are large compared with the inter-flux-line spacing $d(=1.07(\Phi_0/B)^{1/2})$ [2]. Point pins are regions whose dimensions in all directions are less than d, line pins have one dimension long compared with d, surface pins have two dimensions greater than d and volume pins have all dimensions larger than d.

The investigation of the pinning properties in superconductors is usually accomplished by the determination of the volume pinning forces, F_p. As is well known, the volume pinning forces, F_p, is mainly determined by the pinning center size because the pinning center size determines the total length of interacting flux-lines, and the geometrical nature of the interaction. Thus, the magnetic flux pinning classification based on the size of pinning centers is preferred in most cases. However, because the transition of superconducting regions to non-superconducting regions happens in a layer of thickness of coherence length ξ [3,4], which is known as the proximity effect of the superconducting state, those small non-superconducting particles may be induced to become superconducting, and their interaction with the flux-line lattice will not present as 'normal pinning'. Thus the definition of 'core, normal pins' will be invalid in most cases.

In this paper, the imperfection of flux pinning classification based on the pinning center size will be discussed in detail and linear algebra will be employed to investigate the primality of the flux pinning mechanisms.

2. Methods description

Bibby [5] has shown that the pinning strength of non-superconducting particles is independent of the nature of the particles. If non-superconducting metallic particles have a diameter less than or equal to the superconducting coherence length ξ of the matrix, the proximity effect will induce them to become superconducting, and their presence can be regarded as producing a change in the Ginzburg–Landau parameter κ. Those small 'core, normal, point pinning' centers will actually present as 'core, $\Delta\kappa$ pinning' centers. Thus, 'core, normal, point pinning' is believed to be negligible in most cases.

On the other hand, according to Josephson's theory [6–8], two superconductors act to preserve their long-range order across an insulating barrier. With a thin enough barrier, the phase of the electron wavefunction in one superconductor maintains a fixed relationship with the phase of the wavefunction in another superconductor because of the so-called phase coherence. The insulating barrier becomes superconducting but with spatial variations of the Ginzburg–Laudau parameter κ, and then the 'core, normal, surface pinning' will actually present as 'core, $\Delta\kappa$, surface pinning' or 'core, $\Delta\kappa$, volume pinning'. Hence 'core, normal, surface pinning' can be also ignored in real superconducting materials.

Basing on the above discussion, we may ignore 'core, normal, point pinning' and 'core, normal, surface pinning' in the investigation of the pinning properties in superconductors. These ideas can be also applied to Kramer's scaling law [9].

The scaling of volume pinning forces F_p versus the reduced field $h=H/H_{c2}$, where H_{c2} denotes the upper critical field, implies [10,11]

$$F_p = [H_{c2}(T)]^m \times f(h)^n, \quad (1)$$

where m and n are numerical parameters describing the actual pinning mechanism, and $f(h)$ is a function that depends only on the reduced field h. Dew-Hughes model [1] is one of the excellent summation models of the elementary pinning forces. In this model, there are two pinning functions describing the magnetic pinning and six pinning functions describing the core pinning. The latter can be written in a uniform function

$$F_p(h)/F_{p,max} = Ah^p(1-h)^q, \quad (2)$$

where p and q are parameters describing the actual pinning mechanism, and A is a numerical parameter. For various high temperature superconducting materials, the scaling of F_p is found as well, however, experiments have shown that the appropriate scaling field is the irreversibility field H_{irr} instead of H_{c2}, namely $h = H/H_{irr}$ [12–14].

Let us now use linear algebra to investigate the primality of the flux pinning equations of the Dew-Hughes model. Extending the eight pinning functions in the form of h^0, $h^{1/2}$, h^1, $h^{3/2}$, h^2, $h^{5/2}$ and h^3, we obtain

$$f_1 = h^{1/2}(1-2h) = h^{1/2} - 2h^{3/2} = (0,1,0,-2,0,0,0),$$
$$f_2 = h(1-h) = h - h^2 = (0,0,1,0,-1,0,0),$$
$$f_3 = h^{3/2}(1-h) = h^{3/2} - h^{5/2} = (0,0,0,1,0,-1,0),$$
$$f_4 = h^2(1-h) = h^2 - h^3 = (0,0,0,0,1,0,-1),$$
$$f_5 = h^{1/2}(1-h) = h^{1/2} - h^{3/2} = (0,1,0,-1,0,0,0),$$
$$f_6 = (1-h)^2 = 1 - 2h + h^2 = (1,0,-2,0,1,0,0),$$
$$f_7 = h^{1/2}(1-h)^2 = h^{1/2} - 2h^{3/2} + h^{5/2} = (0,1,0,-2,0,1,0),$$
$$f_8 = h(1-h)^2 = h - 2h^2 + h^3 = (0,0,1,0,-2,0,1).$$

In a linear algebra way, we refer the above functions as vectors. It is easy to show that these eight vectors are of dimension seven. Because the number of these vectors (eight) is greater than the number of their dimension (seven), the eight vectors must be linearly dependent [15]. It implies that some of these vectors are not the primal pinning mechanisms but a combination of other pinning mechanisms. We have checked them carefully and find that $f_2 = f_4 + f_8$ and $f_5 = f_3 + f_7$. Thus, two of these functions must be abandoned. Basing on the above discussion, we now throw away 'core, normal, surface pinning, f_7' and 'core, normal, point pinning, f_8'.

As for the appearance of $f_5 = f_3 + f_7$, we may further ascribe it to the fact that, when deducing the specific pinning functions of core interaction, Dew-Hughes has used Gibbs' function [1,16] per unit length of flux-line

$$g = \frac{-\mu_0 \Phi_0 (H_{c2} - H)^2}{2.32(2\kappa^2 - 1)B}. \quad (3)$$

This includes the magnetic and the inter-flux-line interaction terms as well as the core energy, and also the sums of the energy for each flux-line over a radius $d/2$, where d is the inter-flux-line spacing. The core pinning functions f_7 and f_8 may contain parts of the magnetic pinning, thus it causes the appearance of equation $f_5 = f_3 + f_7$.

The pinning force is usually suggested to be zero when no applied field H is present [1]. However, this may be somewhat incorrect. The critical current I_c is at its maximum when no applied field H is present; we call it 'self-field critical current'. When this 'self-field critical current' flows in a superconductor, it will induce a magnetic field. A superconductor that has stronger pinning effect will have a higher self-field critical current, which implies that the pinning force always exists only if a superconductor carries a current, even when no applied field H presents. According to the model of Bean and Livingston [17], the pinning force that causes the self-field critical current, noted as F_{self}, will always be exceeded by the repulsive force and is then independent of the applied magnetic field, which implies that F_{self} can only pin a certain number of flux lines and the remaining flux lines will move to the interior of the superconductor as the applied field H increases.

Let us now pick out F_{self} from the total pinning force and assume that, though somewhat imperfectly, it is approximately independent to the applied magnetic field H, namely

$$F_{self}(h) = \text{constant}. \quad (4)$$

This can be rewritten as

$$F_{self}(h) = a_0 f_0, \quad (5)$$

where a_0 is a constant, and $f_0 = (1,0,0,0,0,0,0)$ represents the pinning force that is independent to the applied magnetic field H.

Because of the complexity of defects in superconductors, several different pinning mechanisms may act simultaneously in them. Wisniewski et al. [18] described the flux pinning of $YBa_2Cu_4O_8$ single crystal by the assumption of two pinning mechanisms. Pu et al. [19] supposed that the total pinning forces in Bi-2223/Ag tapes are the average value of the local pinning forces of each type of the centers. However, for a linear physical system that includes series of element states, can a given state be steadily expressed as the weighted average of the element states in this system? As is known in linear algebra and functional analysis, any vectors in a linear space can be expressed as the weighted average of the elements of a base of this linear space uniquely. According to the above discussion, we now abandon vector f_7 and f_8, and choose the remaining seven vectors f_0, f_1, f_2, f_3, f_4, f_5 and f_6. We extend them in the form of h^0, $h^{1/2}$, h^1, $h^{3/2}$, h^2, $h^{5/2}$ and h^3, and calculate the value of the determinant of these seven vectors

$$\begin{array}{ccccccc} h^0 & h^{\frac{1}{2}} & h^1 & h^{\frac{3}{2}} & h^2 & h^{\frac{5}{2}} & h^3 \end{array}$$
$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 & 0 \end{vmatrix} = 1$$

Because the value of the determinant is not zero, vectors f_0, f_1, f_2, f_3, f_4, f_5 and f_6 are linearly independent. They form a base of the linear space of dimension seven and then they can express any vectors in this linear space uniquely [15]

$$\begin{aligned} F_p(h)/F_{p,\max} &= \sum_{i=0}^{6} a_i f_i \\ &= a_0 + a_1 h^{1/2}(1-2h) + a_2 h(1-h) \\ &\quad + a_3 h^{3/2}(1-h) + a_4 h^2(1-h) \\ &\quad + a_5 h^{1/2}(1-h) + a_6(1-h)^2, \qquad (6) \end{aligned}$$

where $a_i \geqslant 0$. Each term in the right side of Eq. (6) reflects one type of flux pinning mechanism, and parameter a_i reflects the weighting of i type of pinning. In real materials, more than one pinning mechanisms may be cooperative, their effects will add, and then Eq. (6) will be a better description of them.

In general, the pinning force is calculated by equation $F'_p = J_c \cdot B$. However, this equation does not include the pinning force $F_{self}(h) = a_0 f_0$, because $F'_p = 0$ when $B = 0$. Now we define

$$F'_p(h)/F'_{p,\max} = F_p(h)/F_{p,\max} - a_0. \qquad (7)$$

Applying Eq. (7) to the experimental data of Bi-2223/Ag tape [20] in Fig. 1, we find that the main pinning mechanisms are 'core, $\Delta\kappa$, volume pinning f_2', 'core, $\Delta\kappa$, surface pinning f_3' and 'core, $\Delta\kappa$, point pinning f_4'; hence we obtain the exact pinning function

$$\begin{aligned} F'_p(h)/F'_{p,\max} &= 0.73h^{1/2}(1-2h) + h(1-h) \\ &\quad + 1 \cdot 1h^{3/2}(1-h) + 1 \cdot 1h^2(1-h) \\ &\quad + 0.72h^{1/2}(1-h) + 0.24(1-h)^2. \end{aligned} \qquad (8)$$

It should be also kept in mind that the coefficients a_i are not constants, but variables that depend on temperature [11]. Moreover, the pinning analysis is only valid for the true critical current density J_c that is by definition not affected by the flux creep [21,22]. However, for high h (i.e., close to the irreversibility line), the creep effects are most

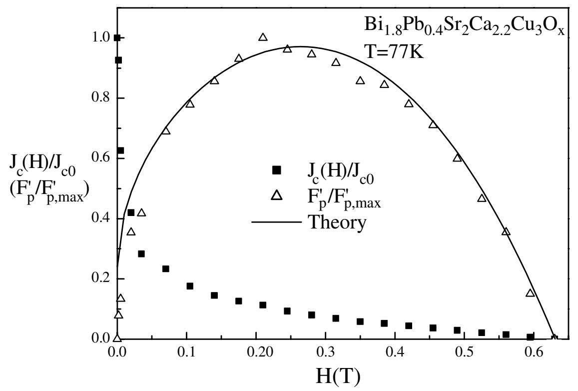

Fig. 1. Normalized pinning force $F'_p(h)/F'_{p,\max}$ vs. the magnetic field H at 77 K with H perpendicular to the Bi-2223/Ag tape surface. The solid line is a theoretical fit for $F'_p(h)/F'_{p,\max}$ with Eq. (7).

important and so the deviations from J_c will be large [23]. At the same time, matching effects and dimensional crossover also rise [24–30]. To account for these, we multiply the right side of Eq. (6) with a term $(1+\mu CkTh^{-n}/U_c)^{-1/\mu}$ that is based on collective pinning theory [31–35], and it yields

$$F_p/F_{p,\max} = (1 + \mu CkTh^{-n}/U_c)^{-1/\mu}\sum_{i=0}^{6} a_i f_i, \qquad (9)$$

where U_c denotes the pinning potential, C is defined by $U[j_s(T),H_a] = kTC$, and k is the Boltzmann constant.

3. Conclusions

In summary, we have investigated the imperfection of flux pinning classification based on the pinning center size. Because of the proximity effect of the superconducting state, 'core, normal, point pinning' and 'core, normal, surface pinning' is suggested to be negligible in most cases. The eight pinning equations in the Dew-Hughes model are found to be linearly dependent by using a linear algebra method. The flux pinning mechanism is reviewed and then seven linearly independent functions were chosen to describe the pinning mechanism in a mathematically self-consistent way. The application of the theory to the experimental data of Bi-2223/Ag tape shows that the theory contributes to a better general description of the complex mechanism of the pinning of the superconductors. Because the calculation and the discussion of the theory are not confined to any specified superconductor crystal structure, the application of the theory to other superconductors is straightforward. It is believed that linear algebra methods can be also used elsewhere to detect if a series of functions form a base of a linear space and thus can describe a physical system perfectly.

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